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Modal Analysis of a Cracked Cantilever Concrete Beam: A Simulation Approach

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Abstract

Cracking in concrete beams remains a critical concern in structural engineering, influencing the safety, durability, and performance of reinforced concrete (RC) elements. This review paper presents a comprehensive analysis of existing research on the modal behavior of cracked concrete beams, focusing on frequency response, displacement characteristics, and von Mises stress variations. The reviewed literature highlights significant advancements in both theoretical and experimental methodologies, including the use of the Finite Element Method (FEM), analytical modeling, and emerging materials such as fiber-reinforced composites and piezoelectric layers for crack detection and control. Studies demonstrate that crack depth, orientation, and location significantly affect natural frequencies and mode shapes, establishing modal analysis as a reliable approach for structural health monitoring. However, existing research reveals gaps in quantitative studies addressing the simultaneous evaluation of modal parameters under cracked conditions. The present work emphasizes the need for integrated FEM-based investigations that can simulate realistic conditions and evaluate multiple response parameters, including frequencies, displacements, and stresses, to improve predictive accuracy in damage assessment. This review serves as a foundation for advancing the application of modal analysis in the diagnosis and rehabilitation of damaged concrete structures.

Keywords: Cracked Concrete Beams, Modal Analysis, Finite Element Method, Structural Health Monitoring, Von Mises Stress, Frequency Response.

1. Introduction

Cracks in reinforced concrete (RC) beams are among the most common forms of structural distress, posing serious implications for safety, durability, and service life. These cracks often originate due to multiple factors such as differential settlement, thermal variations, shrinkage, and external dynamic loads. Understanding their formation and propagation is essential for ensuring the reliability of concrete structures. As reported by Katam et al. (2025), identifying crack location and severity remains a longstanding challenge for structural engineers. Similarly, Utkin and Solovyev (2018) observed that cracks in the tensile zone can alter the neutral axis position, leading to increased stress concentrations and eventual deterioration.

Advancements in construction materials and repair methodologies have substantially improved the ability to mitigate and control cracking. Innovative materials such as Strain-Hardening Cement-based Composites (SHCC) and Steel Fiber Reinforced Concrete (SFRC) have demonstrated superior ductility and crack resistance compared to traditional concrete. Experimental studies by Kim and Yun (2011) and Shen et al. (2022) highlight that SHCC and SFRC significantly delay crack propagation and improve flexural strength. In parallel, epoxy injection techniques (Klym & Blikharskyy, 2023) and Shape Memory Alloys (SMA) (Kuang & Ou, 2008) have emerged as efficient rehabilitation methods, enhancing the structural performance and resilience of damaged beams.

With the growing demand for real-time monitoring and predictive maintenance, modal analysis has become an indispensable tool for evaluating the dynamic behavior of cracked beams. Modal analysis helps determine natural frequencies and mode shapes, both of which are highly sensitive to structural discontinuities. Numerous researchers (e.g., Salmalian et al., 2024; Ma et al., 2022; Khiem, 2022) have established the critical role of modal parameters in detecting and quantifying crack severity. Finite Element Method (FEM)-based studies, such as those by Meshram & Pawar (2015) and Xu et al. (2018), further validate that even minute cracks can induce measurable variations in vibrational responses, underscoring the importance of computational modeling in damage detection.

Recent literature has also explored advanced theoretical frameworks such as Timoshenko, Reddy, and Modified Couple Stress Theories to improve accuracy in modal prediction, particularly for thick or composite beams (Taima et al., 2022; Akbaş, 2017). The integration of smart materials like piezoelectric sensors (Duong et al., 2021) has opened new possibilities for self-sensing structures capable of identifying micro-level cracks before failure occurs. Despite these advancements, a notable gap persists in comprehensive studies that simultaneously analyze frequencies, displacements, and von Mises stresses under cracked conditions.

Hence, the present review paper consolidates existing findings on the modal analysis of cracked concrete beams and identifies critical research gaps. By emphasizing the interrelation between dynamic response parameters and crack-induced structural degradation, this study aims to contribute toward developing more effective numerical models and experimental validation frameworks. Ultimately, understanding the modal behavior of cracked beams will not only enhance diagnostic precision but also facilitate sustainable rehabilitation and maintenance practices in civil infrastructure systems. Following are the objectives of the present research work.

a) Determination of frequencies for different mode shapes;
b) Determination of Von misses stresses for different mode shapes;
c) Determination of displacements for different mode shapes; and
d) Interpretation of results.

2. Literature Review

Research on cracked concrete beams has evolved significantly over the past two decades, with a growing emphasis on understanding their dynamic and modal behavior. Cracks in concrete beams alter stiffness and mass distribution, directly influencing natural frequencies, mode shapes, and vibrational responses. Early analytical studies, such as those by Afshari and Inman (2012), modeled crack behavior using the Rayleigh–Ritz method, representing cracks as rotational springs and demonstrating how crack location and depth affect modal properties. Subsequent finite element analyses by Meshram and Pawar (2015) and Liu et al. (2012) further established the correlation between crack parameters and frequency shifts, confirming that even small cracks produce measurable changes in dynamic characteristics.

Later research incorporated more sophisticated modeling techniques and theoretical frameworks. Salmalian et al. (2024) and Pei (2024) applied the Finite Element Method (FEM) to analyze cracked fiber metal laminated and functionally graded beams, emphasizing FEM's accuracy in simulating vibrational responses. Ma et al. (2022) and Khiem (2022) extended this work by analyzing nonlinear behaviors caused by crack orientation and stiffness variations, concluding that both geometric asymmetry and local flexibility strongly influence modal frequencies. Experimental investigations by Siva and Ramakrishna (2020) and Khalkar et al. (2022) validated these numerical findings, confirming that modal parameters can serve as reliable indicators of damage in cracked beams. These studies established modal testing as a practical approach for early-stage damage detection in reinforced concrete structures.

Advancements in beam theory have also improved the accuracy of vibration modeling. Taima et al. (2022) used Timoshenko and Reddy beam theories to demonstrate that higher-order models capture shear effects neglected in Euler–Bernoulli analysis. Similarly, Akbaş (2017) applied Modified Couple Stress Theory for microscale beams, showing that material gradation and scale effects significantly impact modal responses. The adoption of nonlinear formulations by Chajdi et al. (2017) and Pešić et al. (2015) highlighted the limitations of traditional linear models, suggesting that nonlinear dynamic analysis better represents the actual behavior of cracked concrete beams.

Material innovation has also played a vital role in the evolution of modal analysis. Duong et al. (2021) incorporated piezoelectric layers into cracked beam systems to improve structural health monitoring. Their findings showed that variations in piezoelectric layer thickness affect both vibration frequencies and electric charge output, supporting the feasibility of self-sensing structures. Xu et al. (2018) analyzed steel–concrete interface damage and observed reductions in natural frequencies proportional to the severity of cracking. Likewise, Zhou and Liu (2016) examined dynamic responses under moving loads, revealing that repetitive vehicular stresses accelerate crack propagation and affect beam stiffness and reliability.

Efforts to develop computational and reliability-based models have further refined the field. Gao and Xie (2024) proposed time-dependent reliability formulations that integrate stiffness degradation over time, improving dynamic load assessments. Optimization algorithms developed by Moezi et al. (2015) also enhanced crack identification accuracy, demonstrating that computational techniques can significantly improve diagnostic precision. Together, these studies have strengthened the understanding of how structural cracks influence dynamic performance, offering predictive tools for maintenance and safety evaluation.

Despite extensive progress, several research gaps remain. Most existing works analyze either frequency or displacement but rarely address von Mises stresses simultaneously with modal properties. Experimental validation under varied crack configurations is still limited, and there is little integration between modal analysis and long-term reliability studies. Furthermore, while the incorporation of smart materials shows potential, large-scale implementation for real-time monitoring remains in its infancy. Addressing these gaps could substantially enhance the predictive and diagnostic capabilities of modal analysis for reinforced concrete structures.

Overall, the reviewed literature demonstrates that combining advanced numerical modeling, experimental validation, and intelligent sensing technologies offers a promising pathway toward accurate and efficient structural health monitoring. Modal analysis continues to serve as a critical bridge between theoretical understanding and practical assessment of cracked concrete beams, enabling safer and more durable engineering designs.

The following points represent the gaps in the research:

a) There were very limited number of research papers found which were focused on investigations on frequencies, displacements and von misses stresses; and
b) There were also very limited numbers of research papers found which were focused on studying the cracked beams under these investigated properties.

3. Solution Methodology

In the present research work, modal analysis of the beam was targeted, which was accomplished on CATIA software, the details of which are presented as follows:

CATIA V5 (Computer Aided Three-dimensional Interactive Application), developed by Dassault Systèmes, is one of the most powerful and widely used platforms for computer-aided design (CAD), engineering (CAE), and manufacturing (CAM). It provides an integrated environment where design, analysis, and visualization can be carried out seamlessly within a single software interface.

In engineering applications, CATIA V5 is extensively used for 3D modeling, simulation, and structural analysis. The software allows engineers to design complex components, assemblies, and products, and to analyze their behavior under various mechanical and environmental conditions. Its Generative Structural Analysis (GSA) module supports static, thermal, and modal analyses, enabling the evaluation of stresses, displacements, and natural frequencies before physical prototypes are built.

The primary advantage of CATIA V5 lies in its integration of design and analysis. Once a model is created, boundary conditions, material properties, and loads can be defined directly on the same geometry. This integration reduces errors that typically occur when transferring models between separate design and analysis software. CATIA's finite element analysis (FEA) capabilities make it possible to discretize the model into elements, apply loads and constraints, and compute stresses and deformations accurately.

In the context of modal analysis, CATIA V5 allows the determination of natural frequencies and mode shapes of structures, which are essential for understanding dynamic behavior and avoiding resonance. The results include graphical representations of mode shapes and color-mapped contours for displacement and von Mises stress distribution, making it easier to visualize regions of high stress or vibration amplitude.

Another notable strength of CATIA V5 is its user-friendly interface and parametric modeling. Any design modification is automatically reflected in the analysis, ensuring consistency throughout the workflow. The software's precision, efficiency, and interoperability with other platforms such as ANSYS, SolidWorks, and Abaqus make it an indispensable tool in both academic research and industrial design.

In summary, CATIA V5 provides a comprehensive solution for modeling, simulation, and analysis, allowing engineers to validate the structural performance of components under realistic conditions. Its ability to combine design and modal analysis within one environment enhances productivity, accuracy, and innovation in engineering research and development.

And with the help of the software, the following properties were investigated:

a) **Frequencies**

The natural frequencies are the specific vibration rates at which a structure tends to oscillate when it is disturbed.

b) **Displacements**

Displacement is the measure of the change in position of a point on the structure from its original location during vibration.

c) **Von Mises Stresses**

Von Mises stress is the equivalent stress used to predict yielding of materials under complex loading conditions.

4. Case Study

The section is devoted to the details or problem formulation and solution, the details of which are presented in upcoming sub-sections.

4.1 Problem Formulation and Solution

The following sub-sections present the details of the problem formulation and its solution.

4.1.1 Problem Formulation

The following is the stepwise procedure of problem formulation.

a) First of all a cantilever beam of the following dimensions was designed.

Table 4.1: Dimensions of Cantilever Beam

S.No	Property	Value (mm)
1.	Length	600
2.	Width	40
3.	Depth	15

b) In the next step, triangular notches of height 5 mm and base length 4 mm were inserted into the beams at spans 200 mm and 400 mm from the fixed ends. Figure 4.1 shows the details of the cantilever beam.

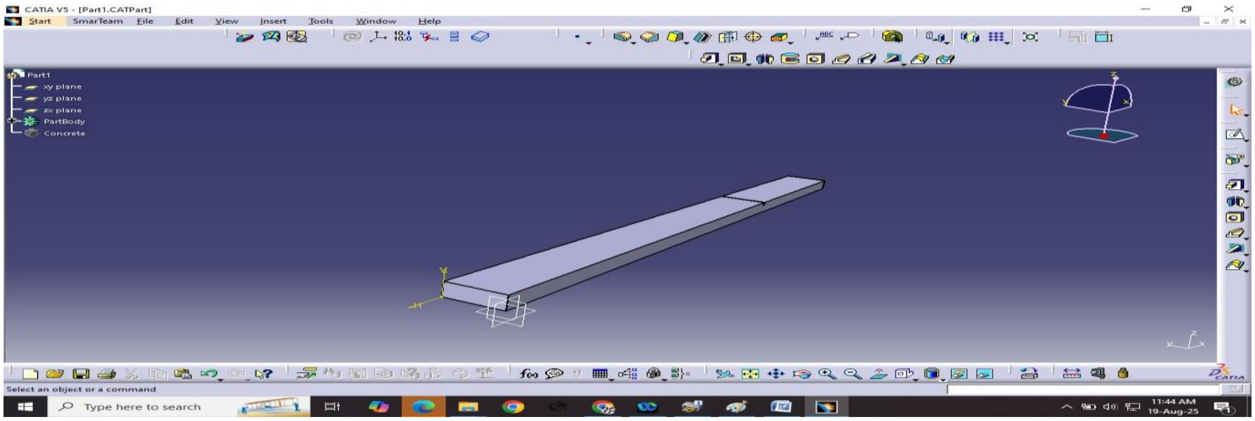


Figure 4.1: Model of Cantilever Beam

4.1.2 Problem Solution

In order to solve the problem, the following stepwise procedure was adopted.

a) In the first step of investigating the solution, concrete M20 was assigned as beam material.

b) In next step, meshing of the models was performed. Objective of meshing is to make a body deformable due to which it can respond to applied changes in properties external to it. Table 4.3 shows the details of meshing parameters.

Table 4.3: Details of Meshing Parameters

S.No	Property	Value
1.	Number of nodes	18267
2.	Number of elements	3457
3.	Type of elements	Octree tetrahedron
4.	Element size	30 mm

Figure 4.2 presents the meshed model of the beam.

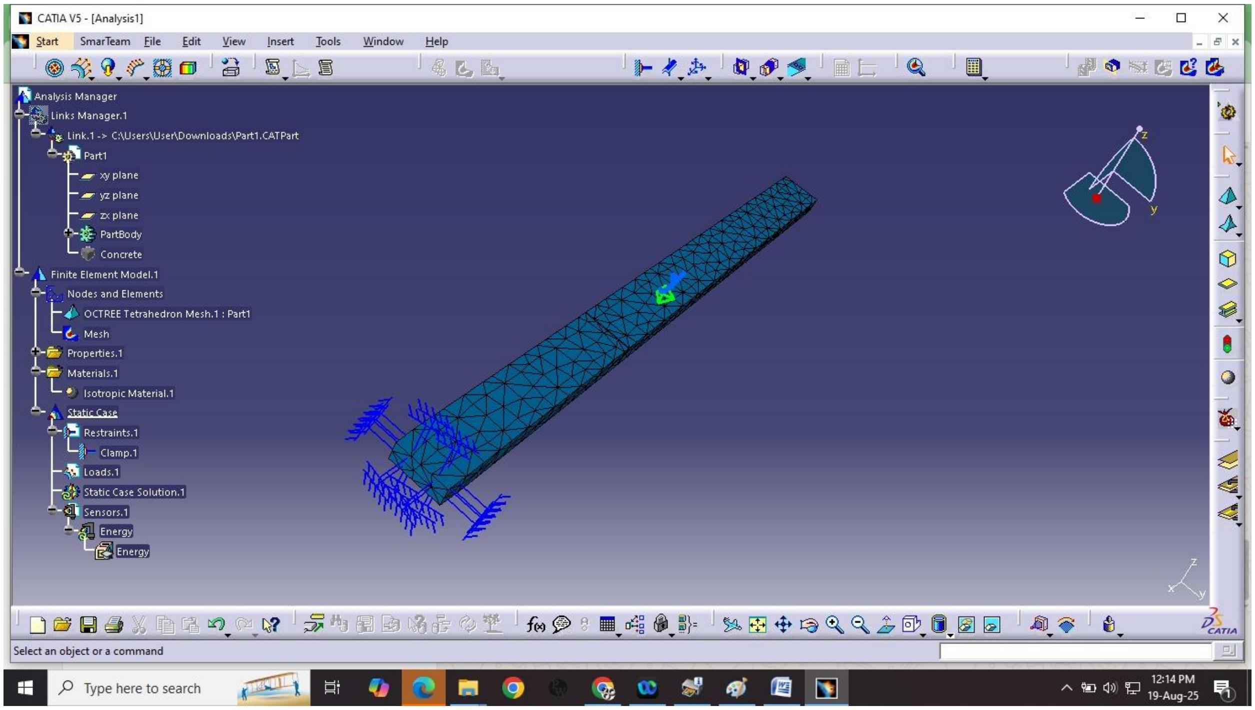


Figure 4.2: Meshed Model of Beam

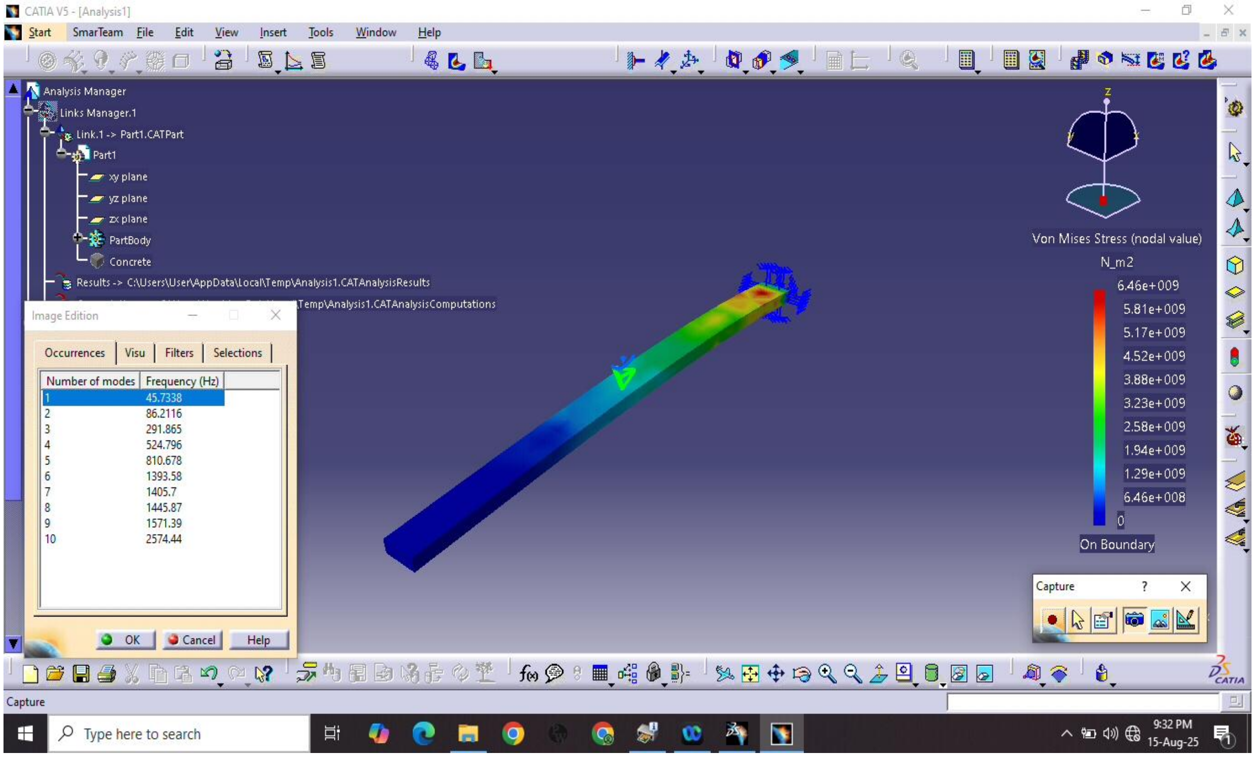
c) In the last step of research work, one end of the beam was made fixed and modal analysis was performed, during which frequencies, Von misses stresses as well as displacements for different mode shapes were investigated, the details of which are presented in upcoming chapter.

5. Results and Discussion

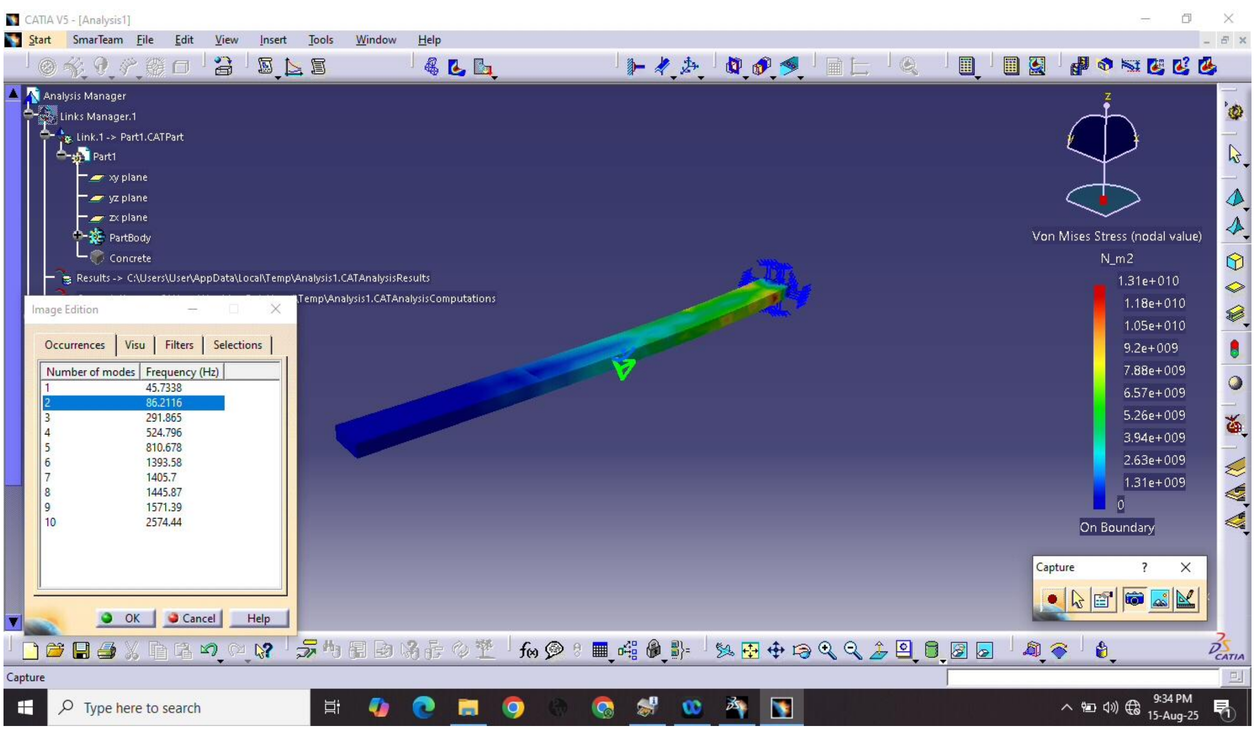
The present section is devoted to the resutls obtained and discussion made about the results, the details of which are presented in upcoming sub-sections.

5.1 Results

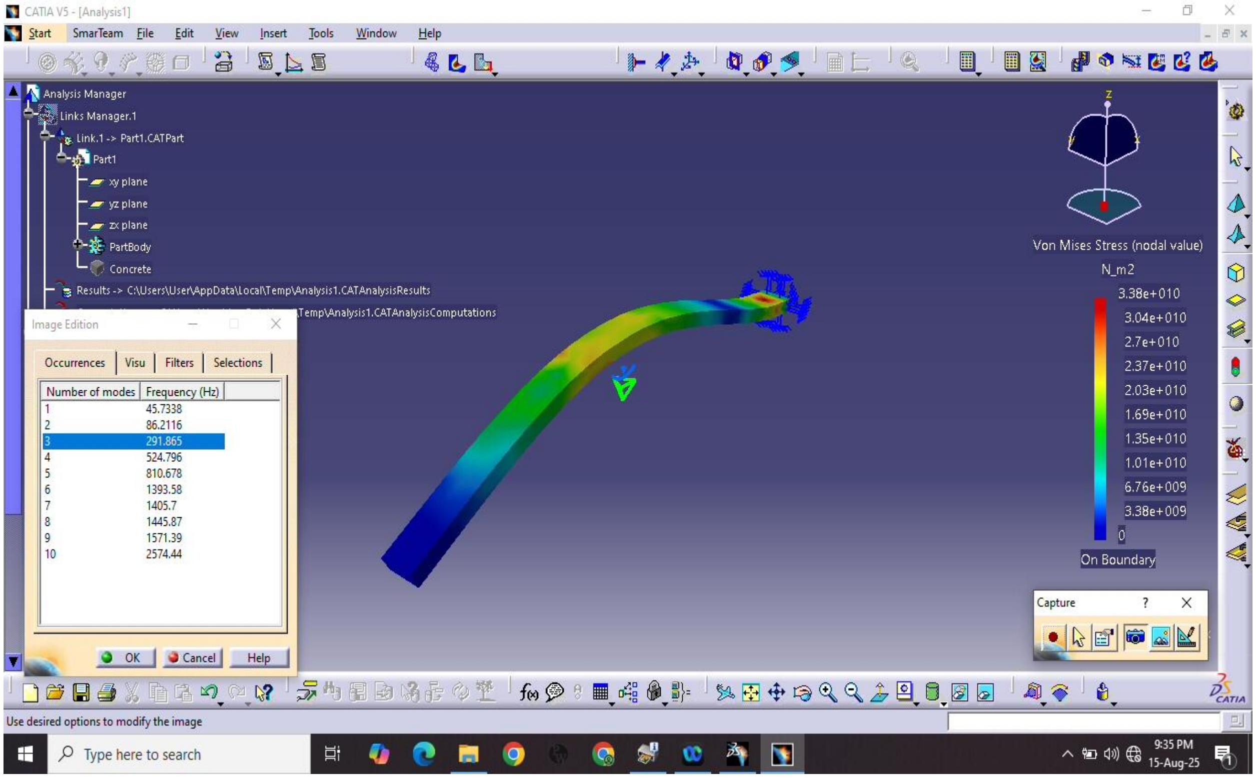
Figure 5.1 to Figure 5.4 present the results of research work for different properties showed by different cracked beams.



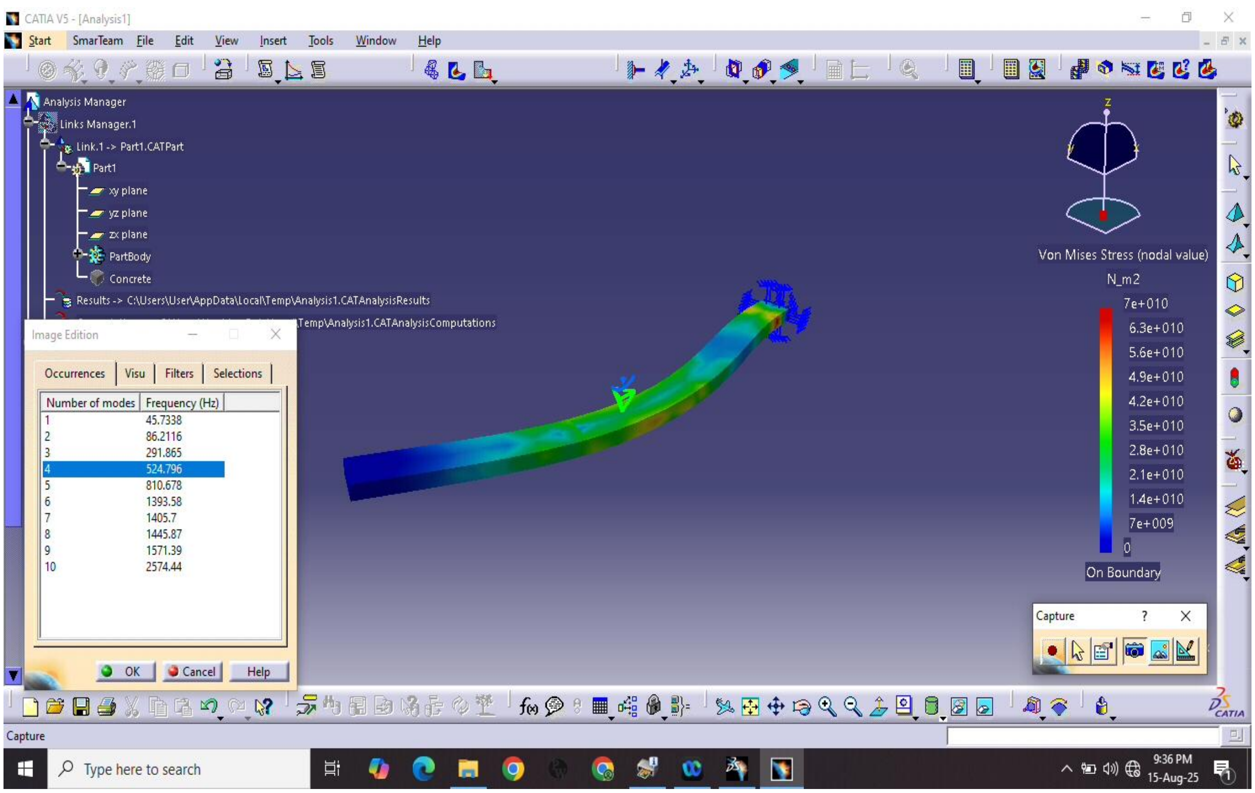
a) Mode shape 1



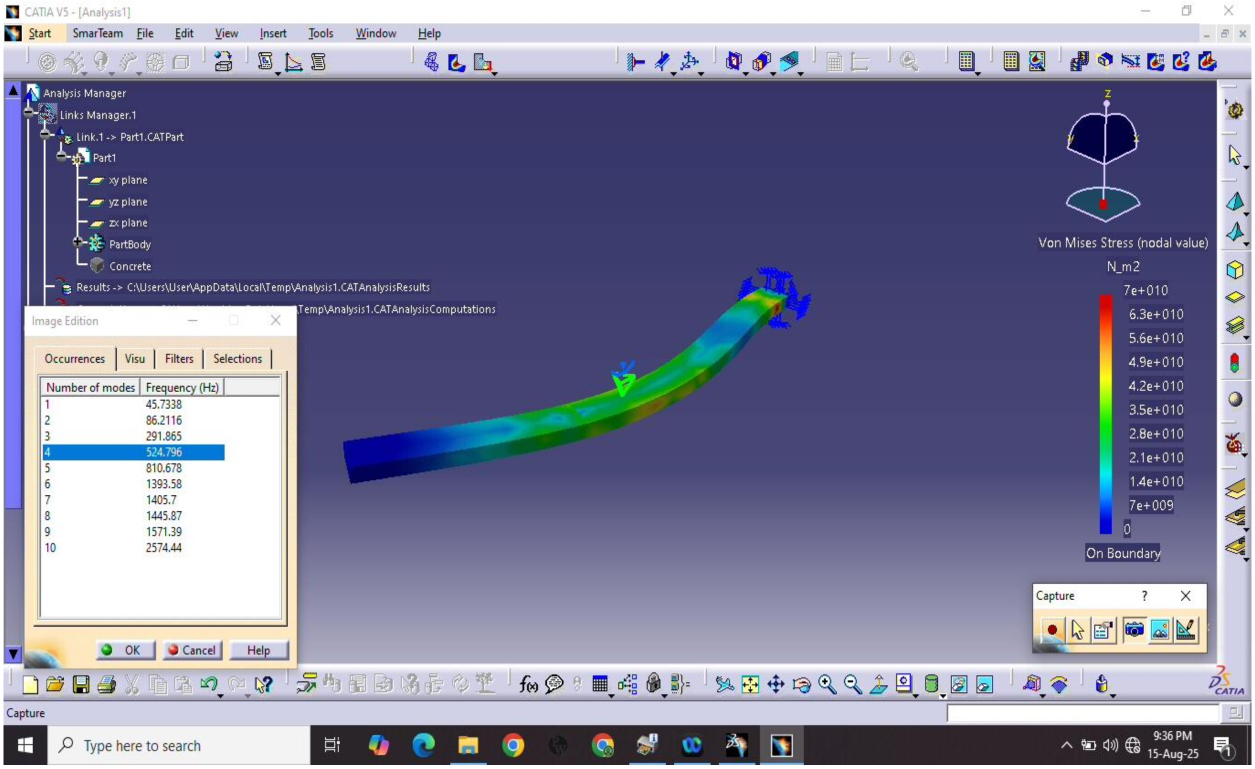
b) Mode shape 2



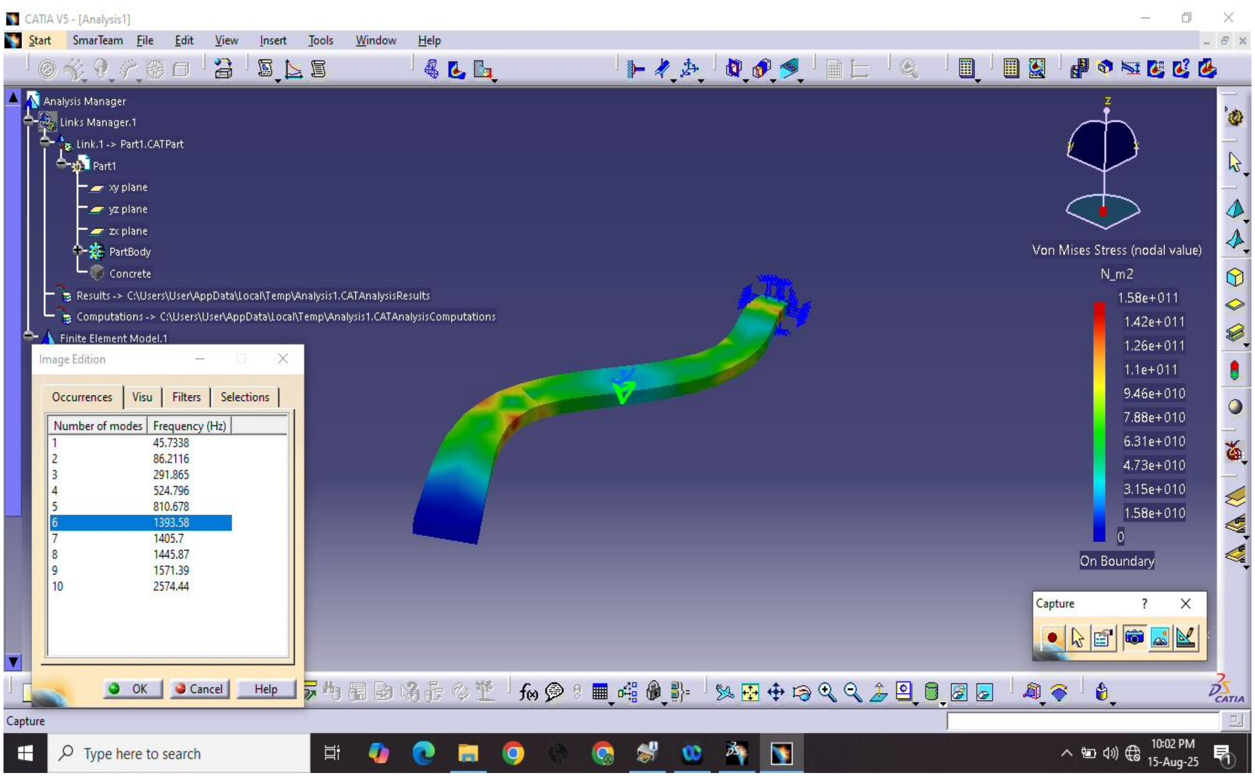
c) Mode shape 3



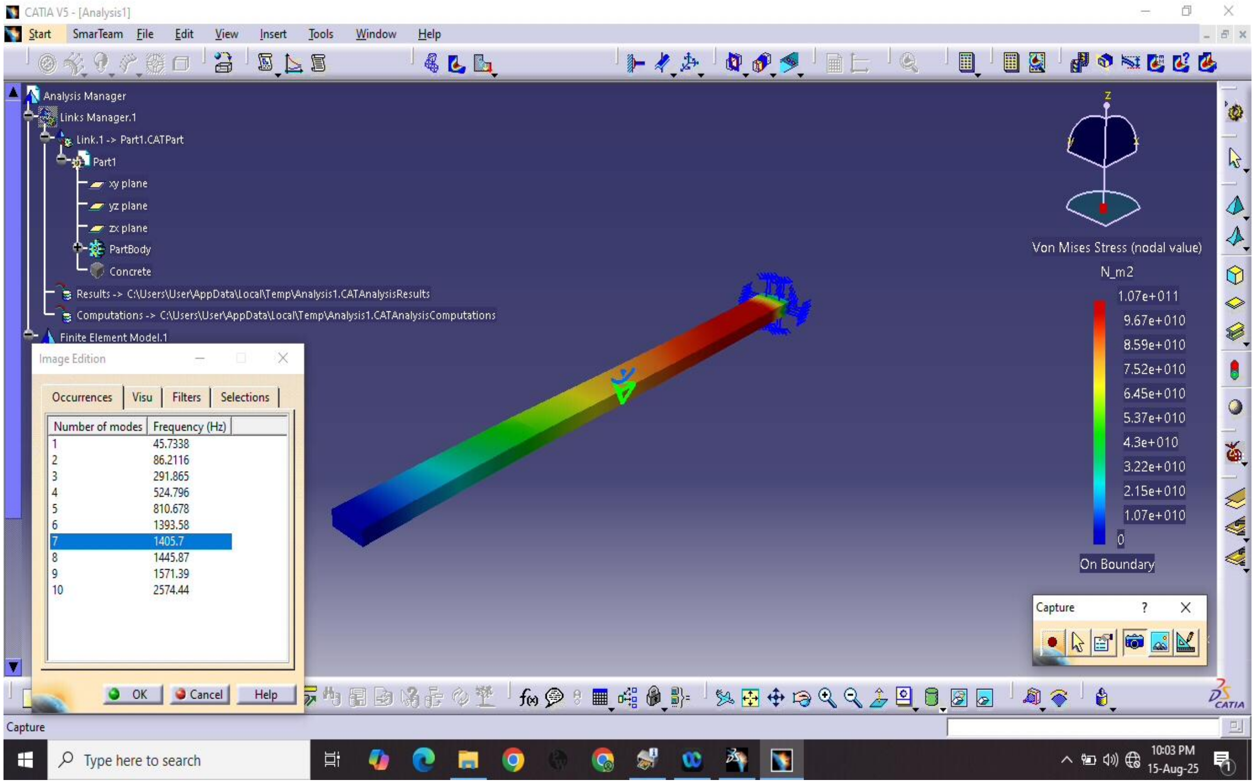
d) Mode shape 4



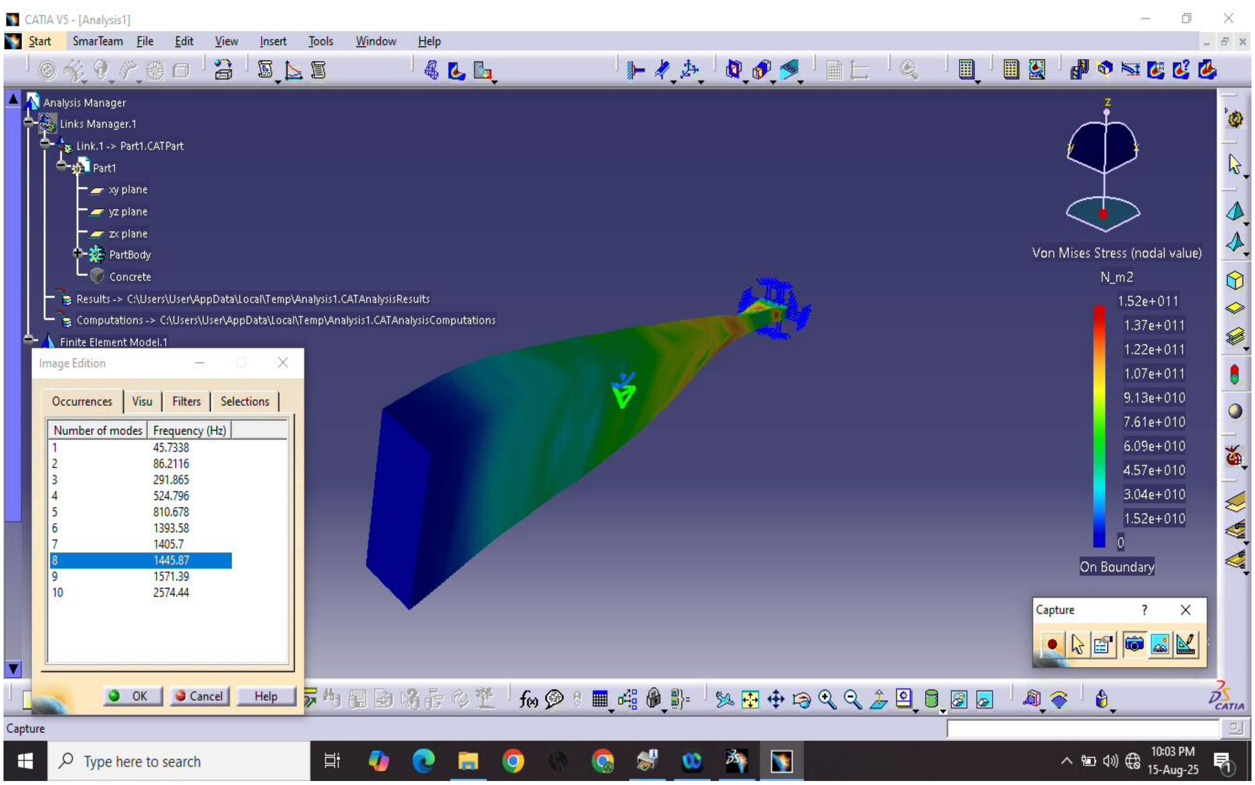
e) Mode shape 5



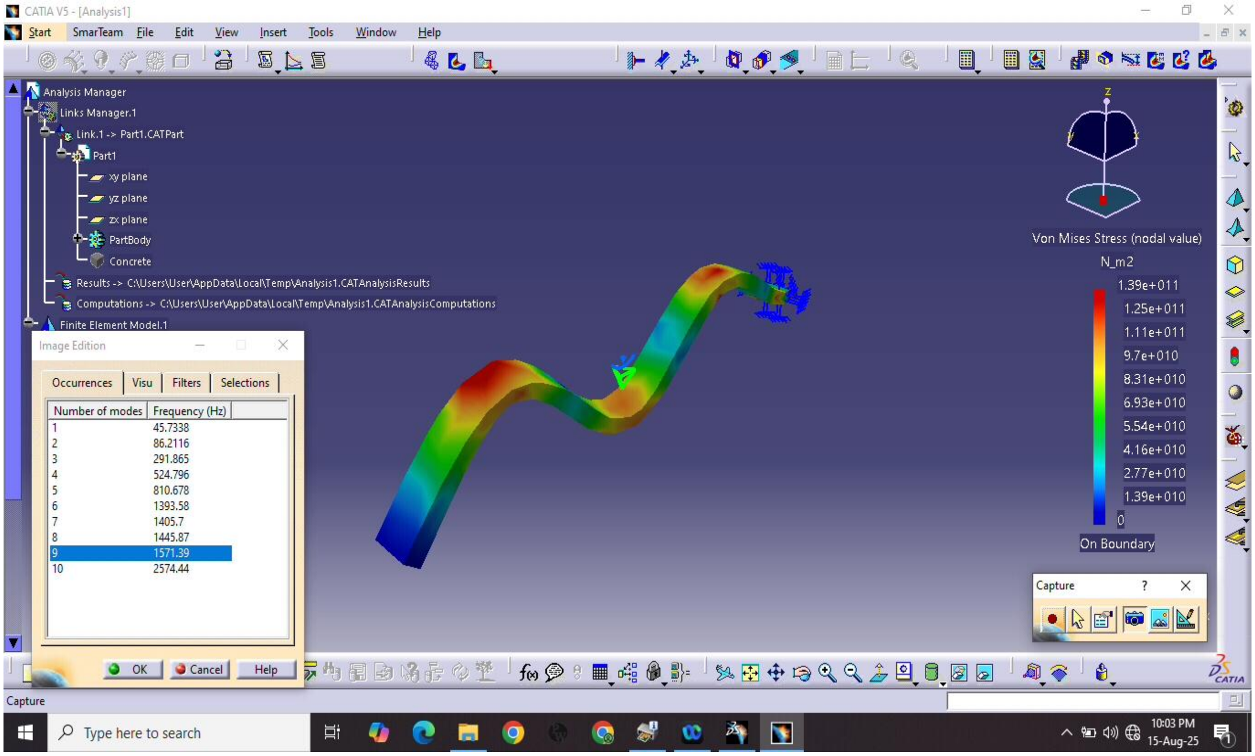
f) Mode shape 6



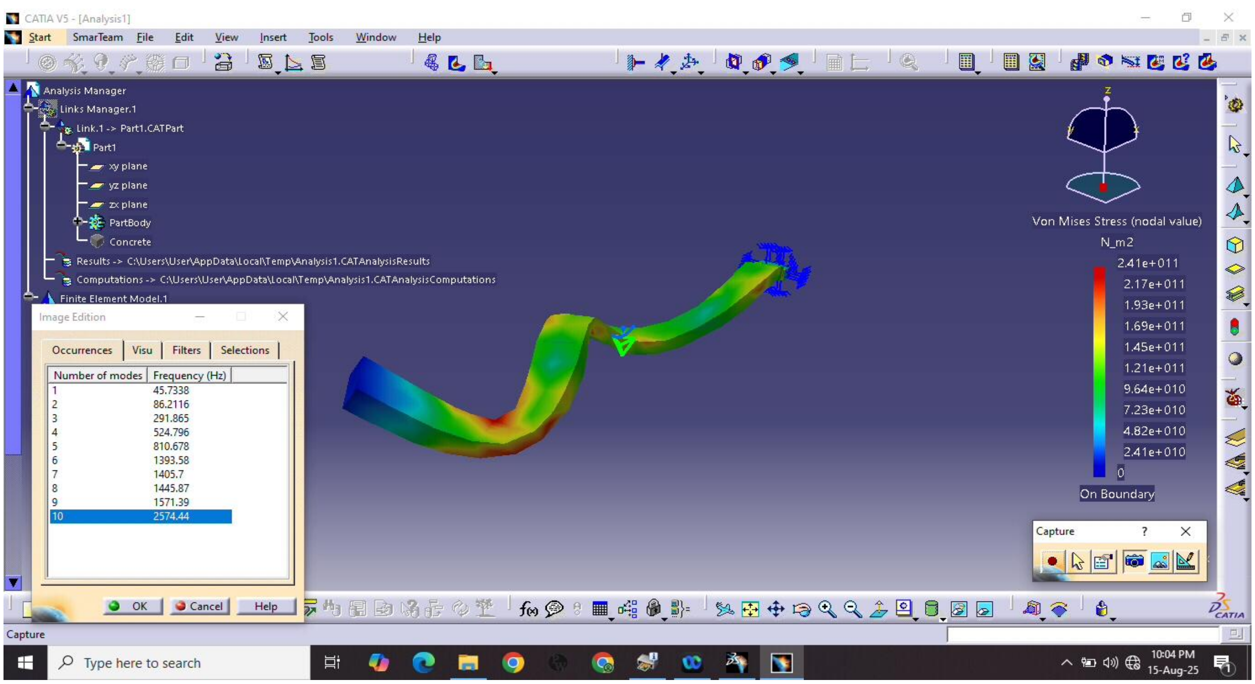
g) **Mode shape 7**



h) **Mode shape 8**

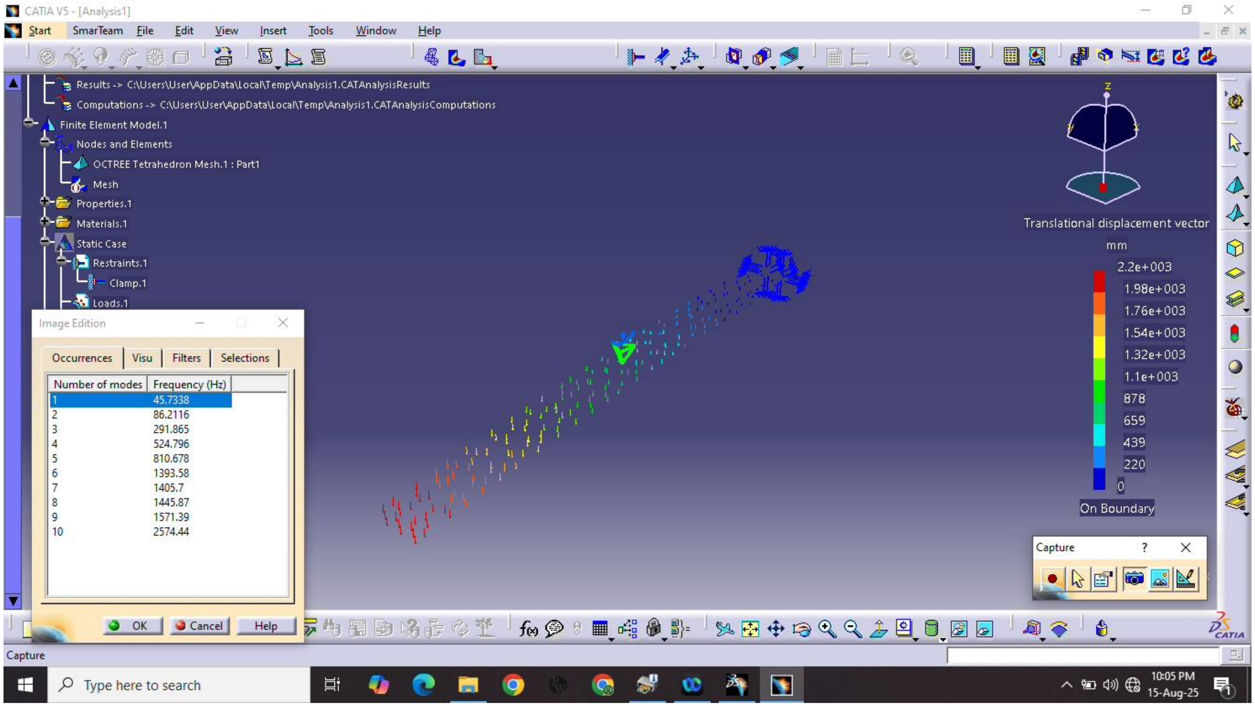


i) Mode shape 9



j) Mode shape 10

Figure 5.1: Von misses stresses and Frequencies at crack location 200 mm from the fixed end



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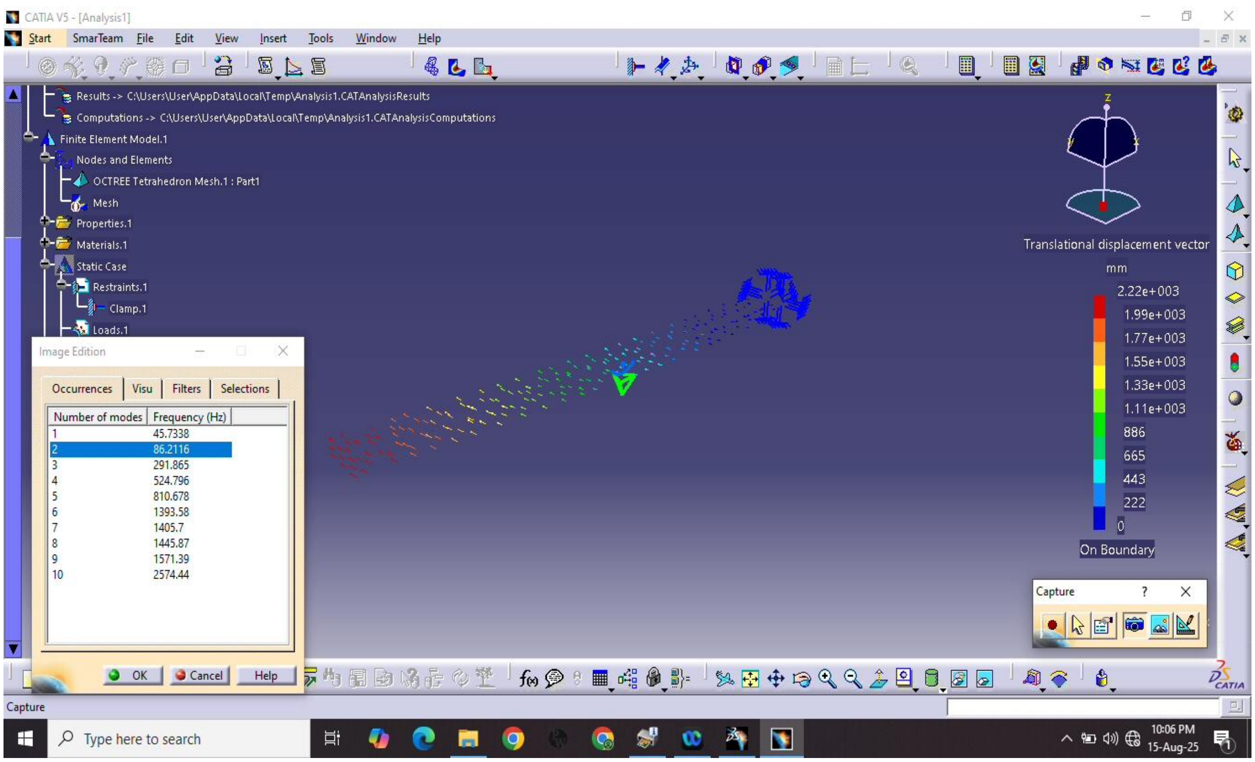

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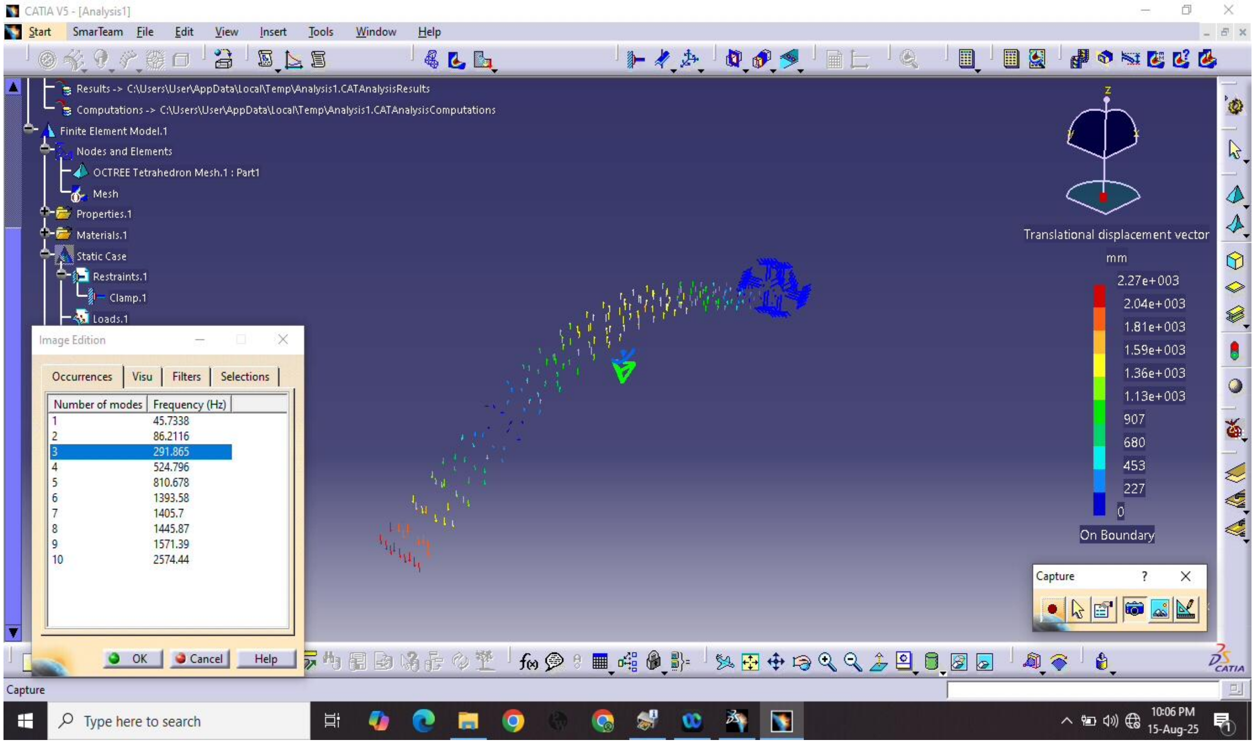

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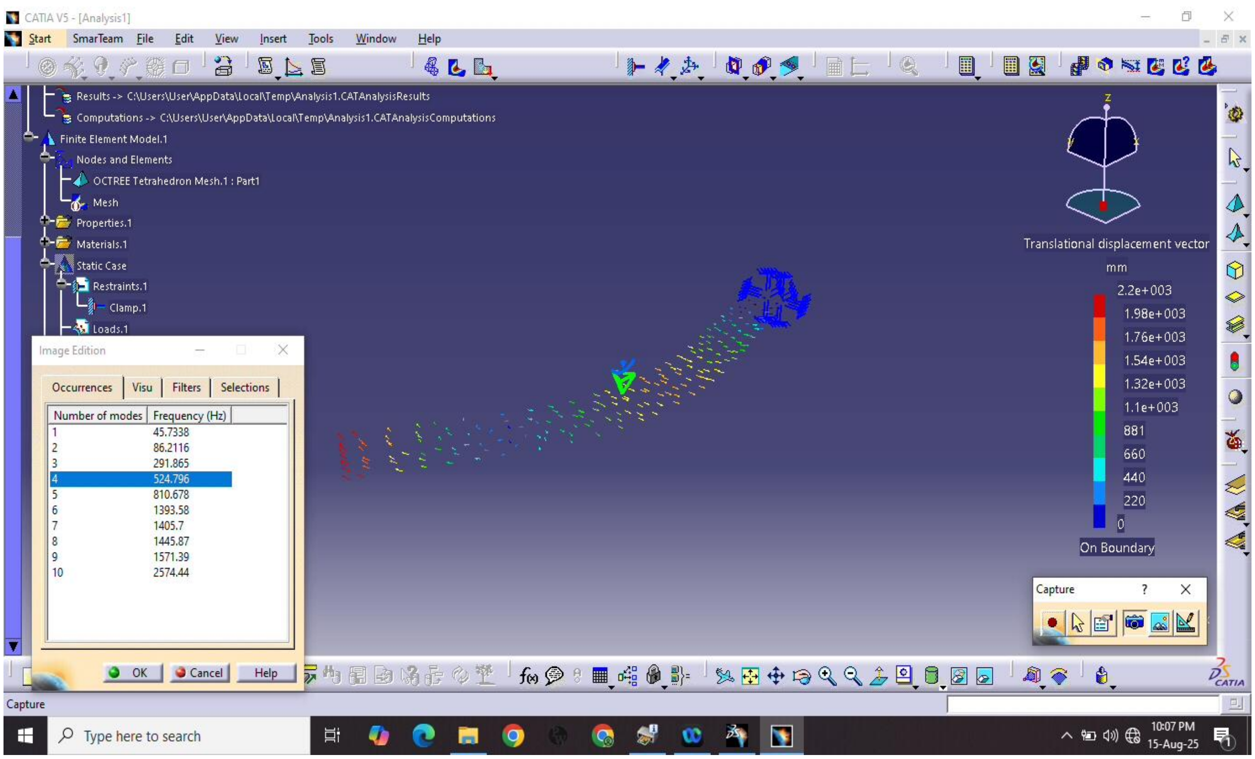

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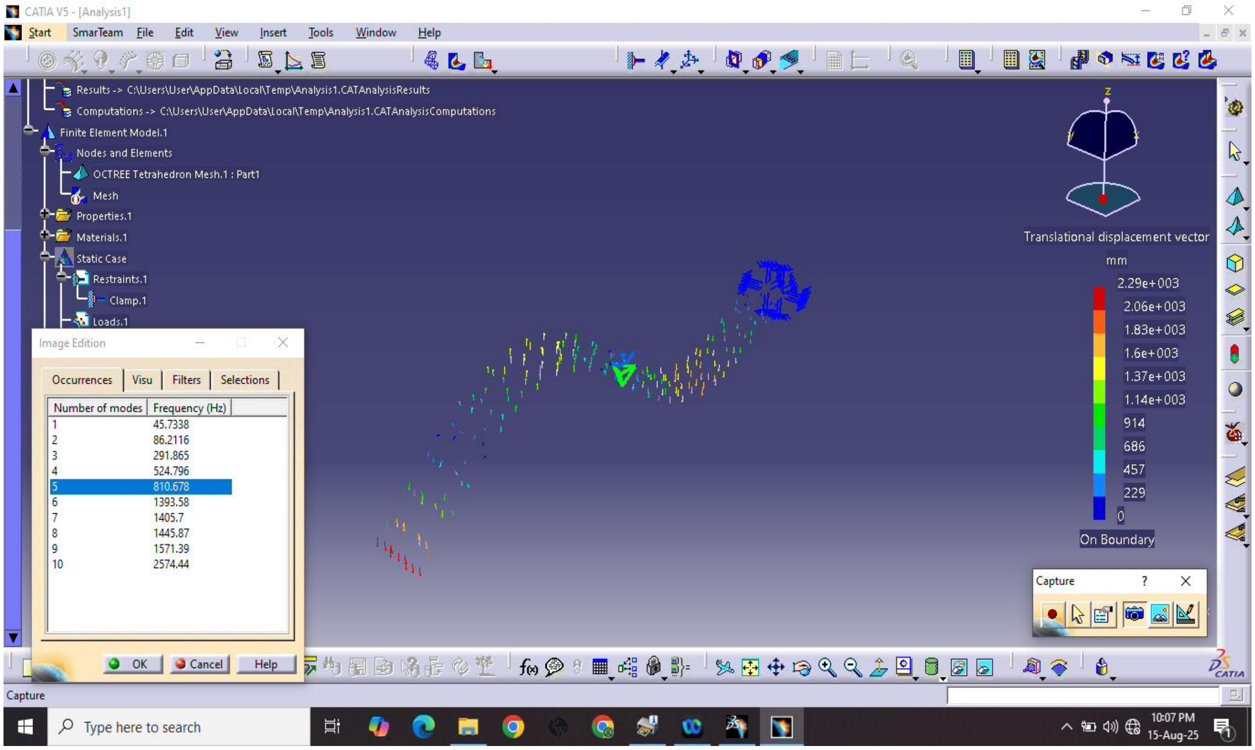

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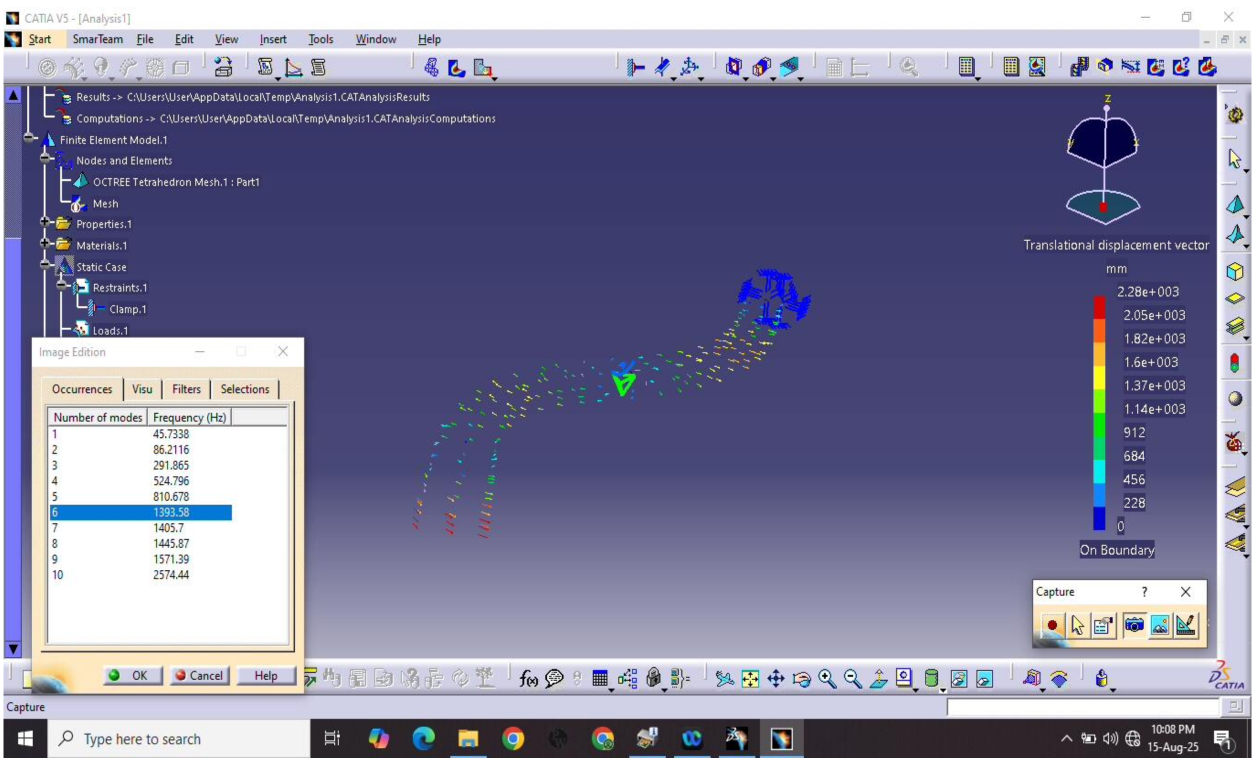

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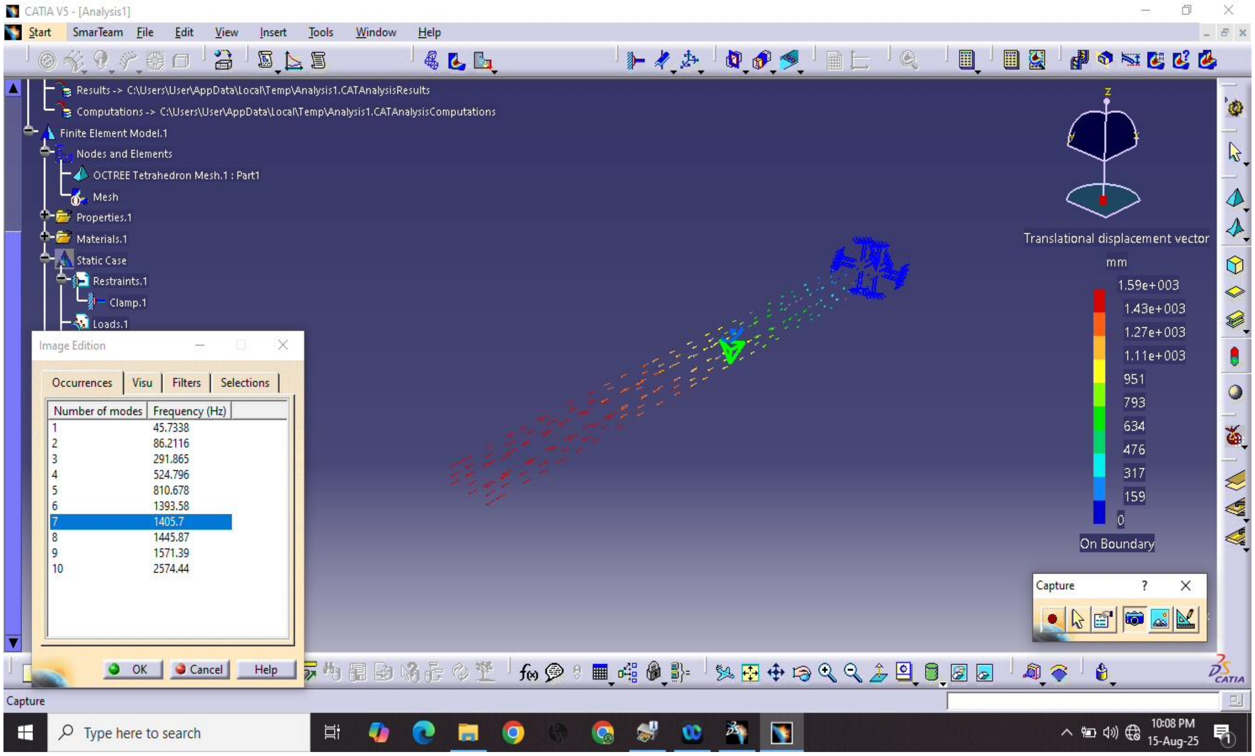

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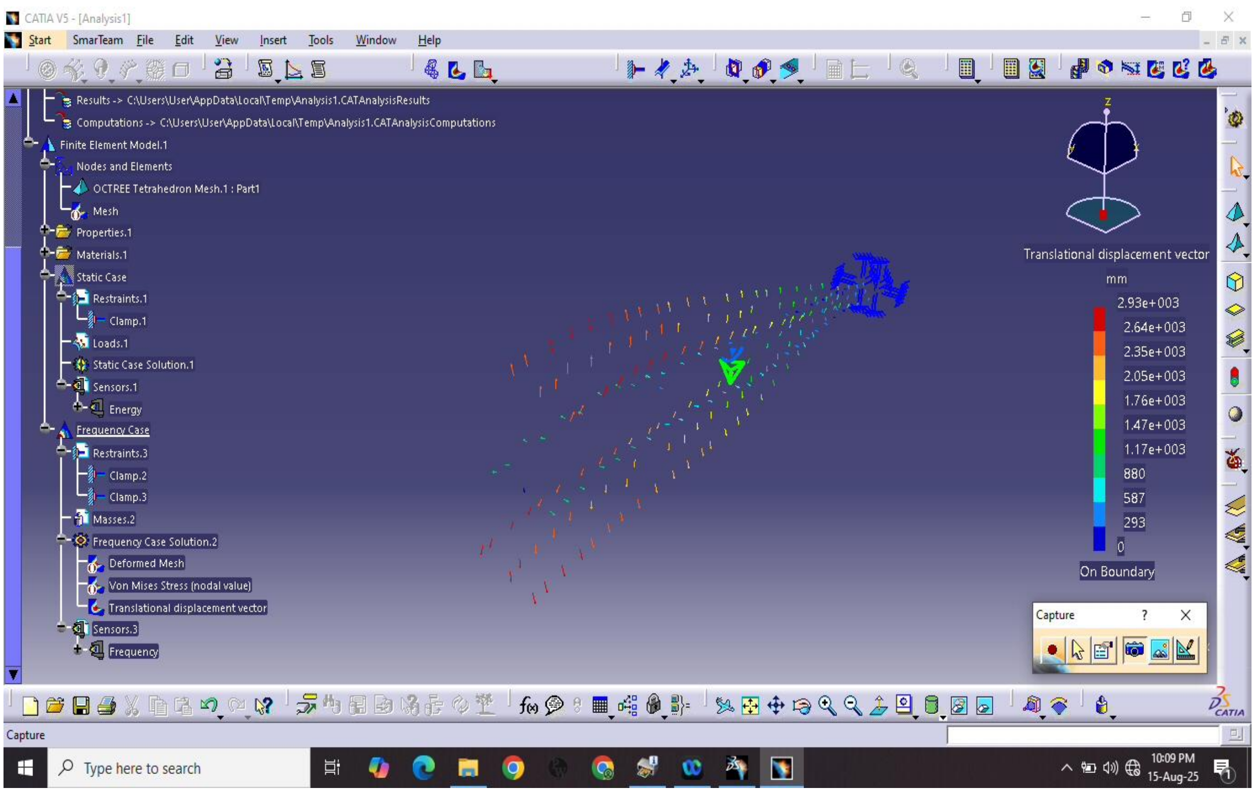

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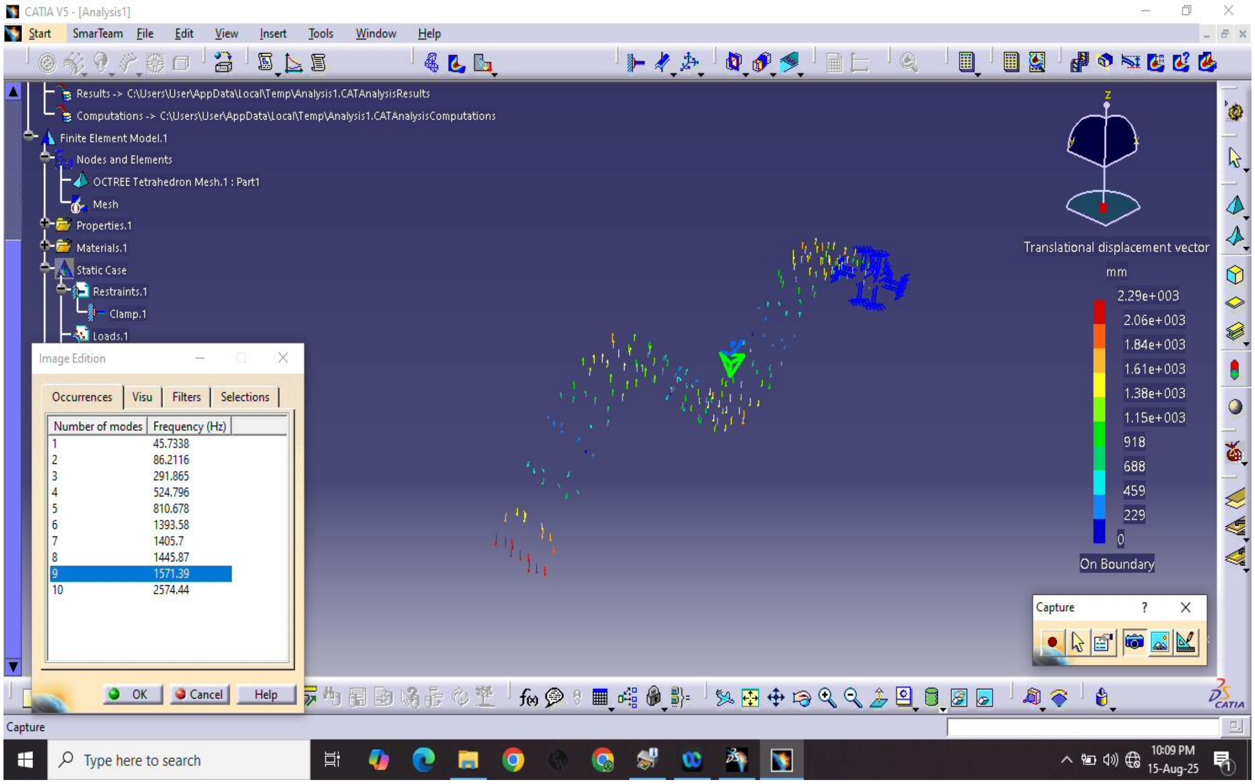

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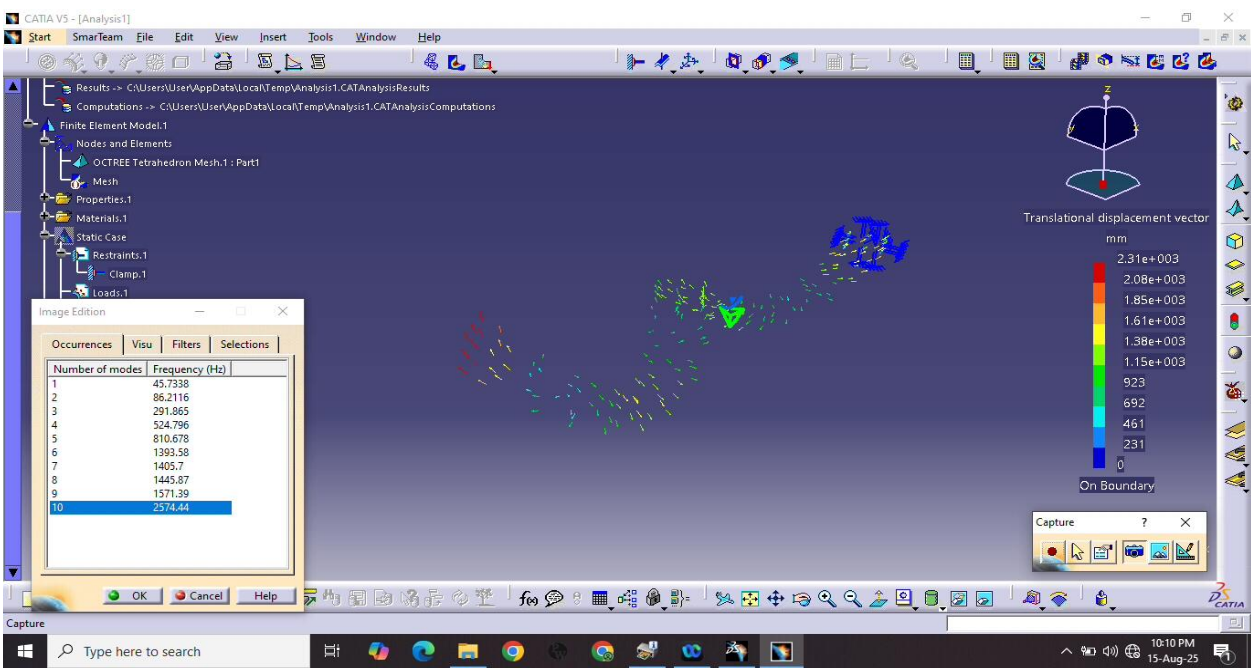

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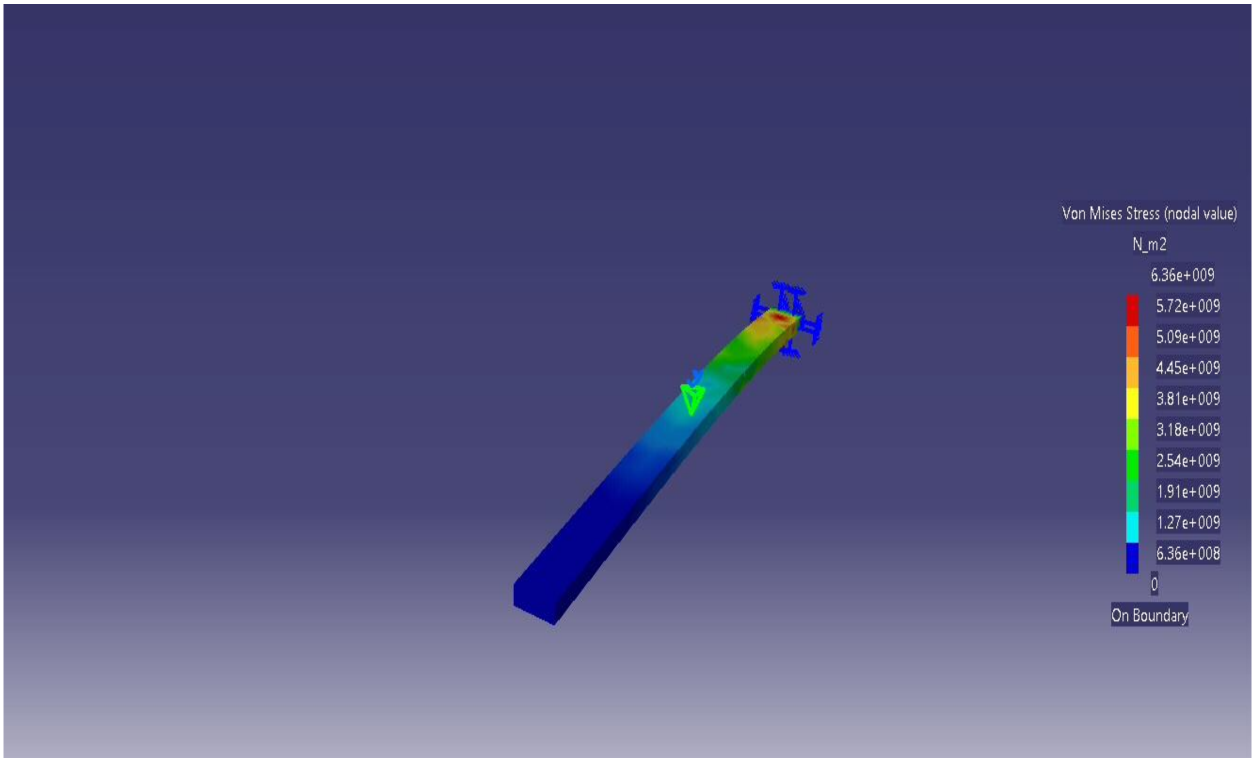
Figure 5.2: Displacements and Frequencies at crack location 200 mm from the fixed end



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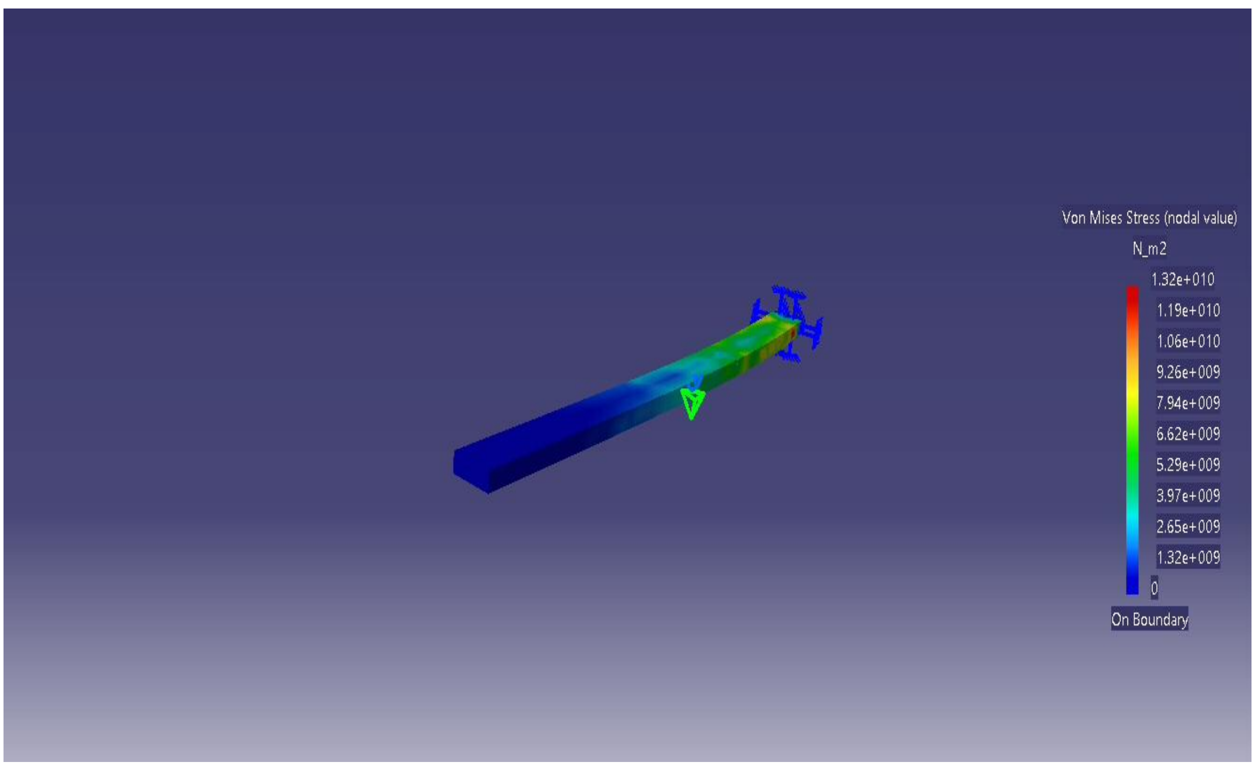

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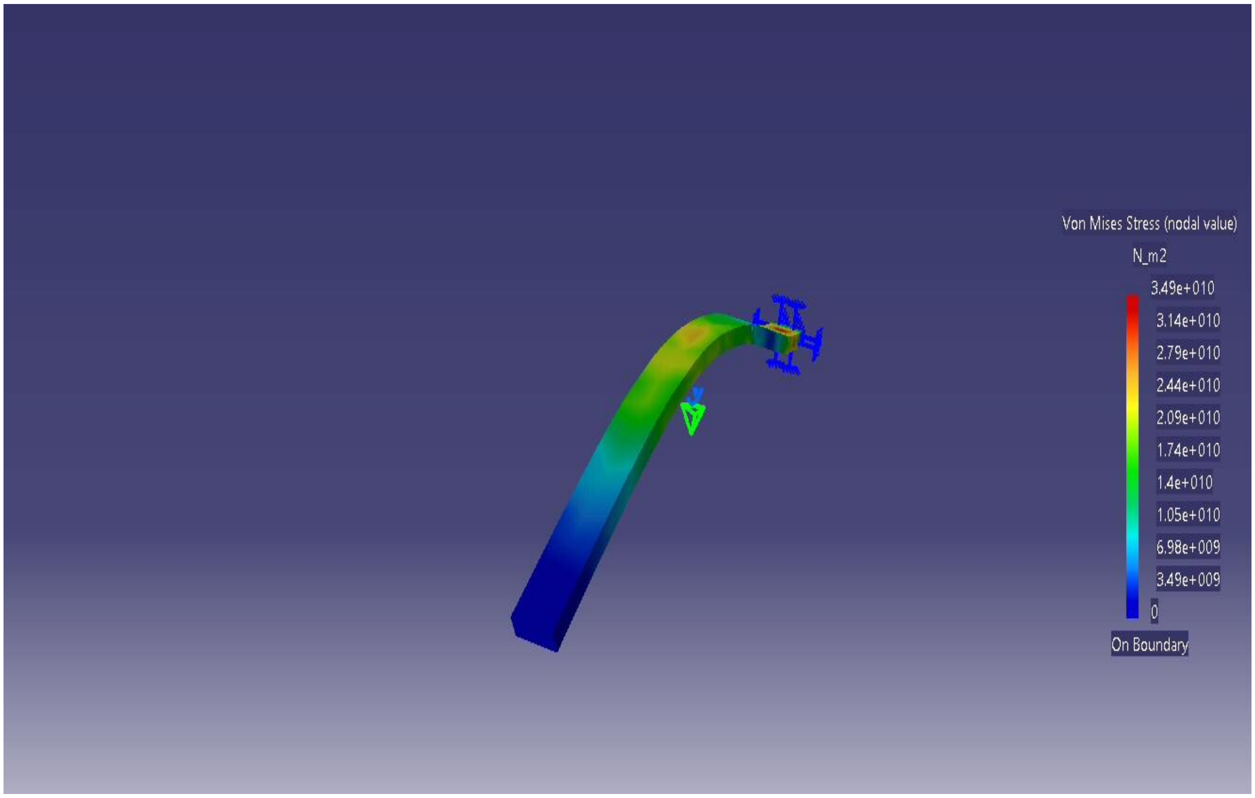

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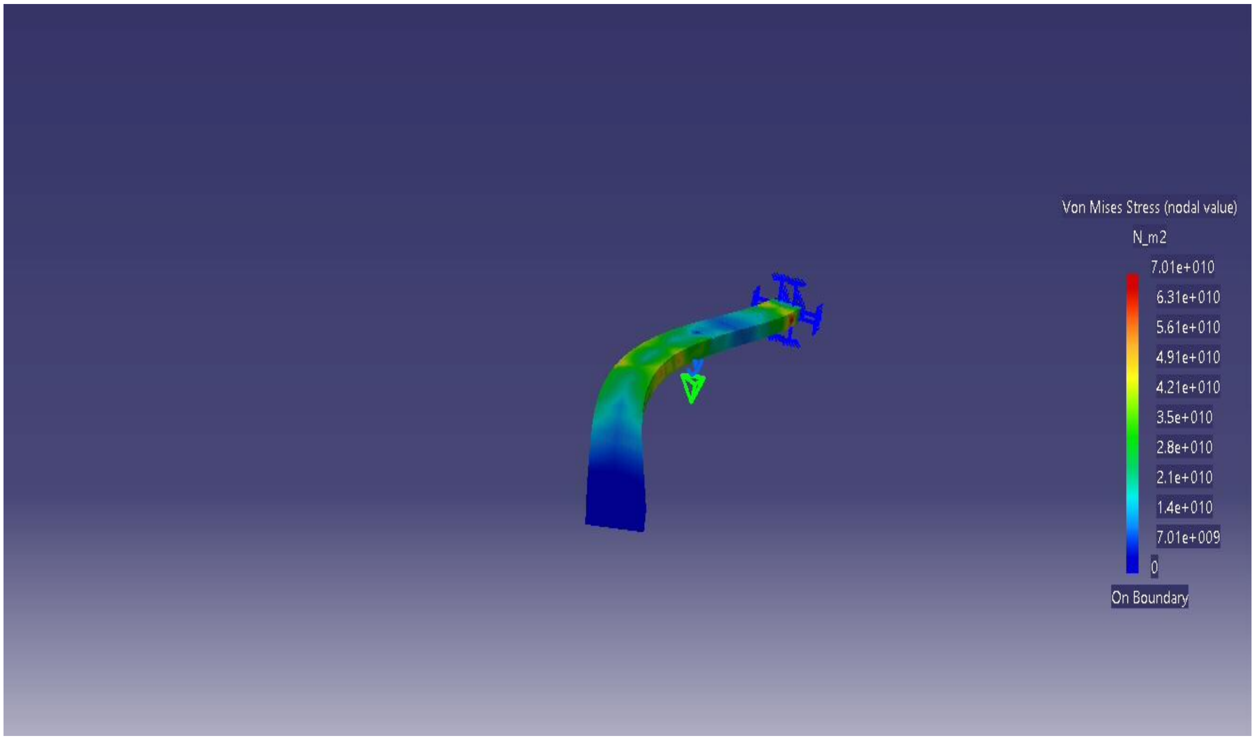

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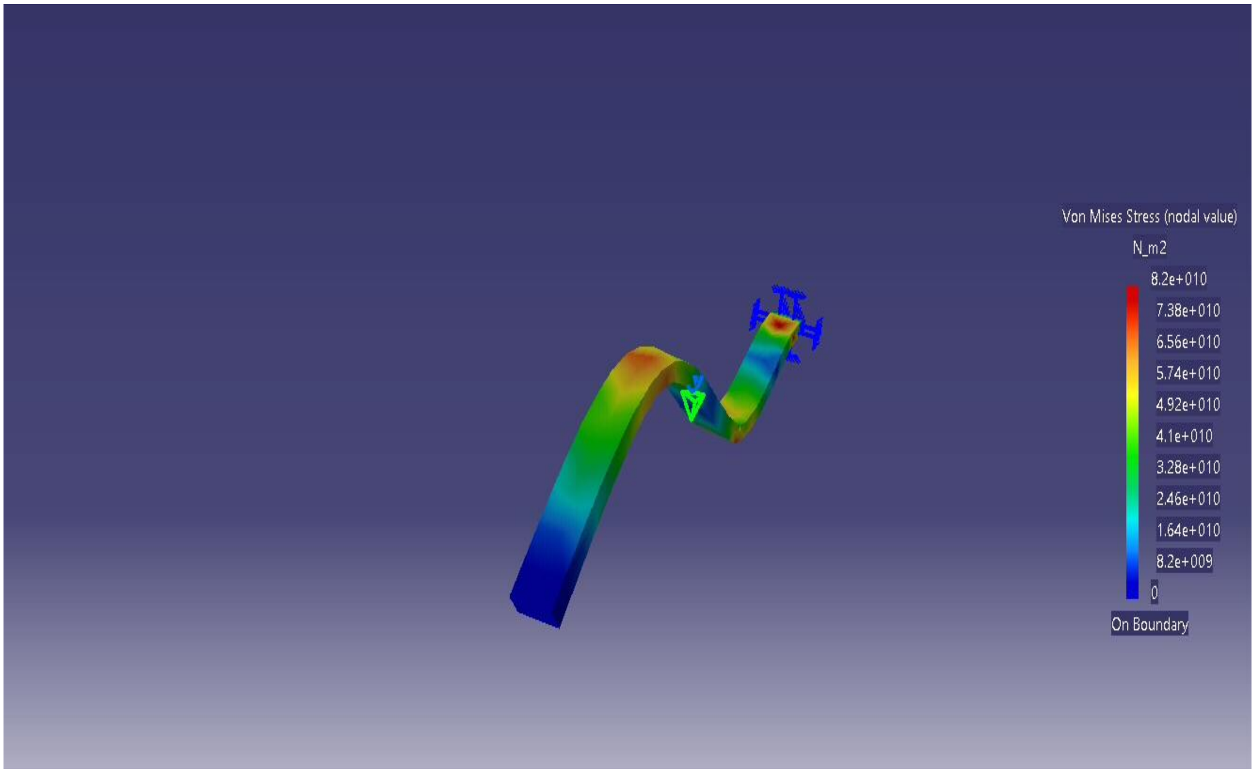

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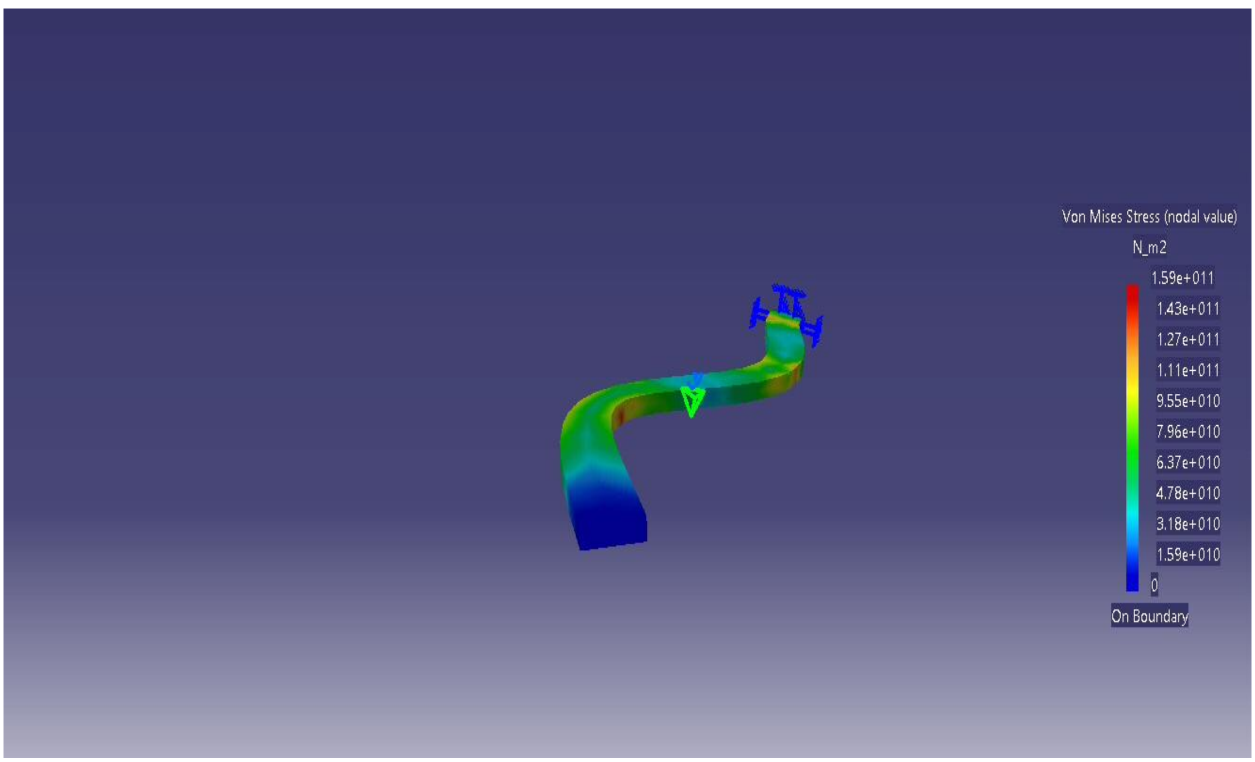

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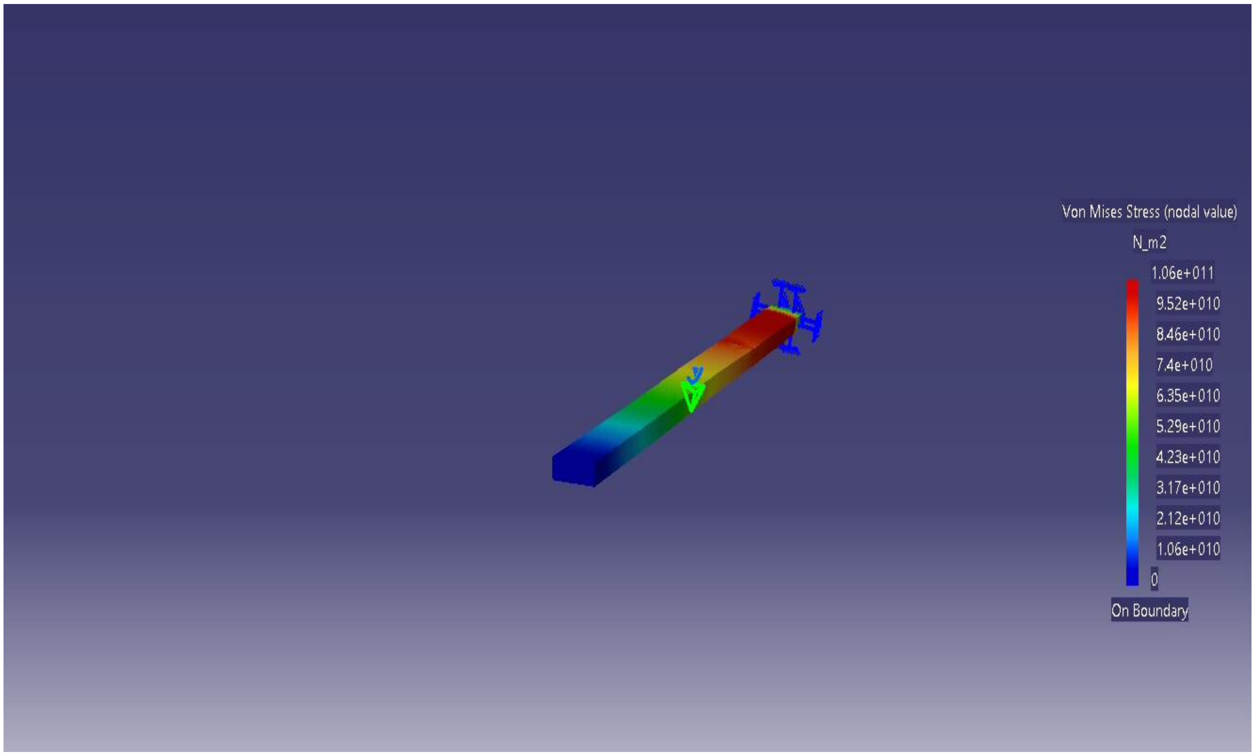

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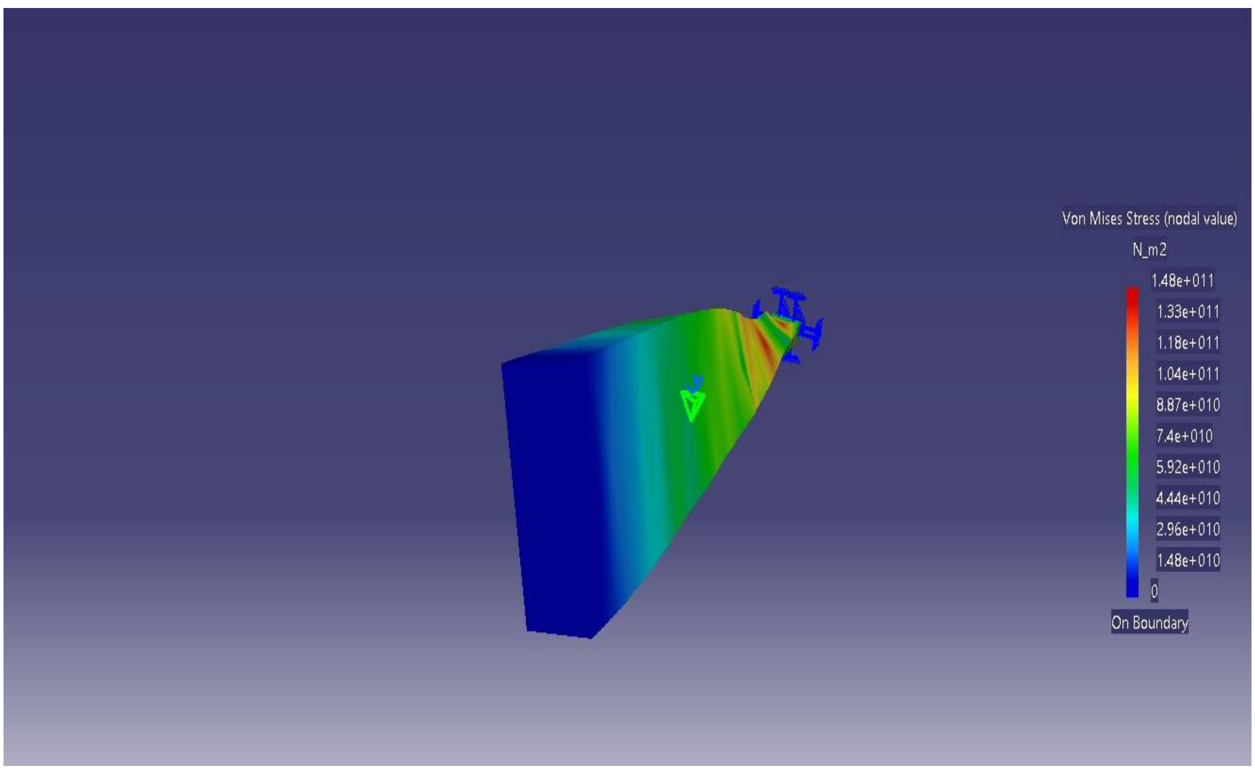

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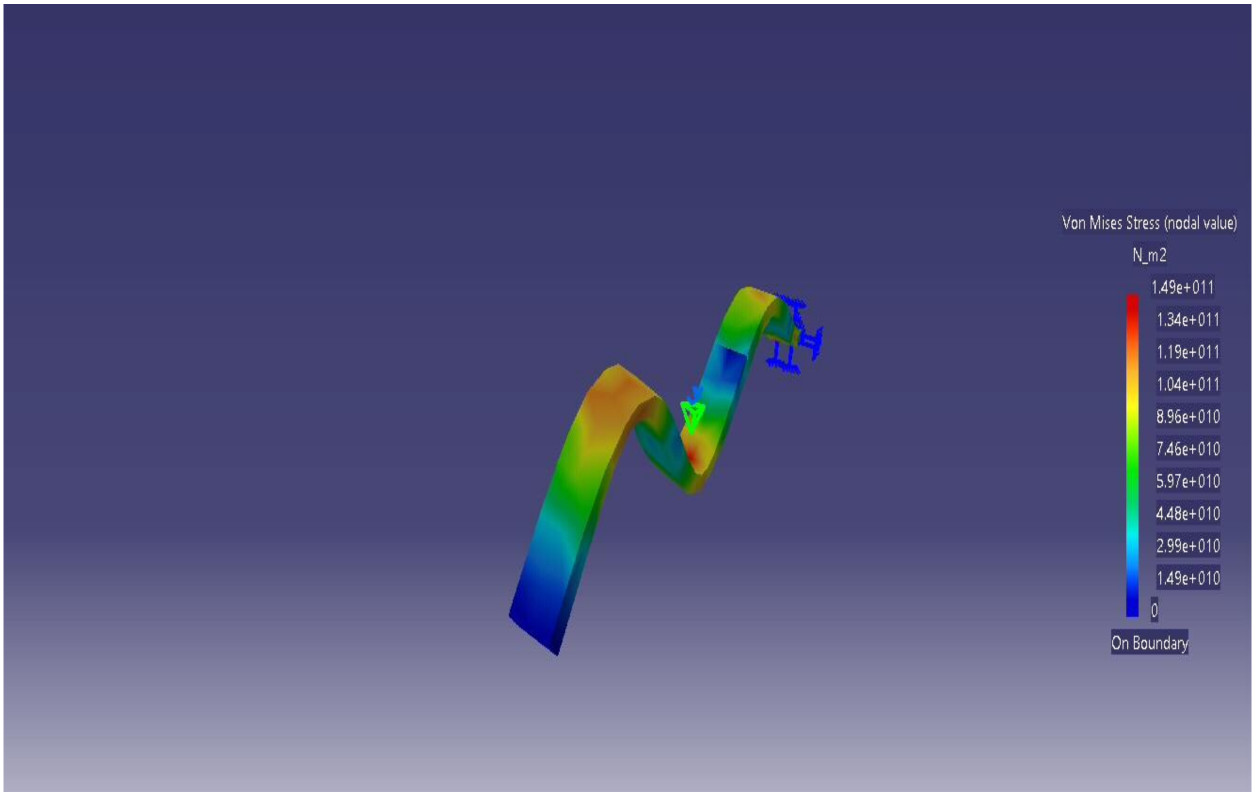

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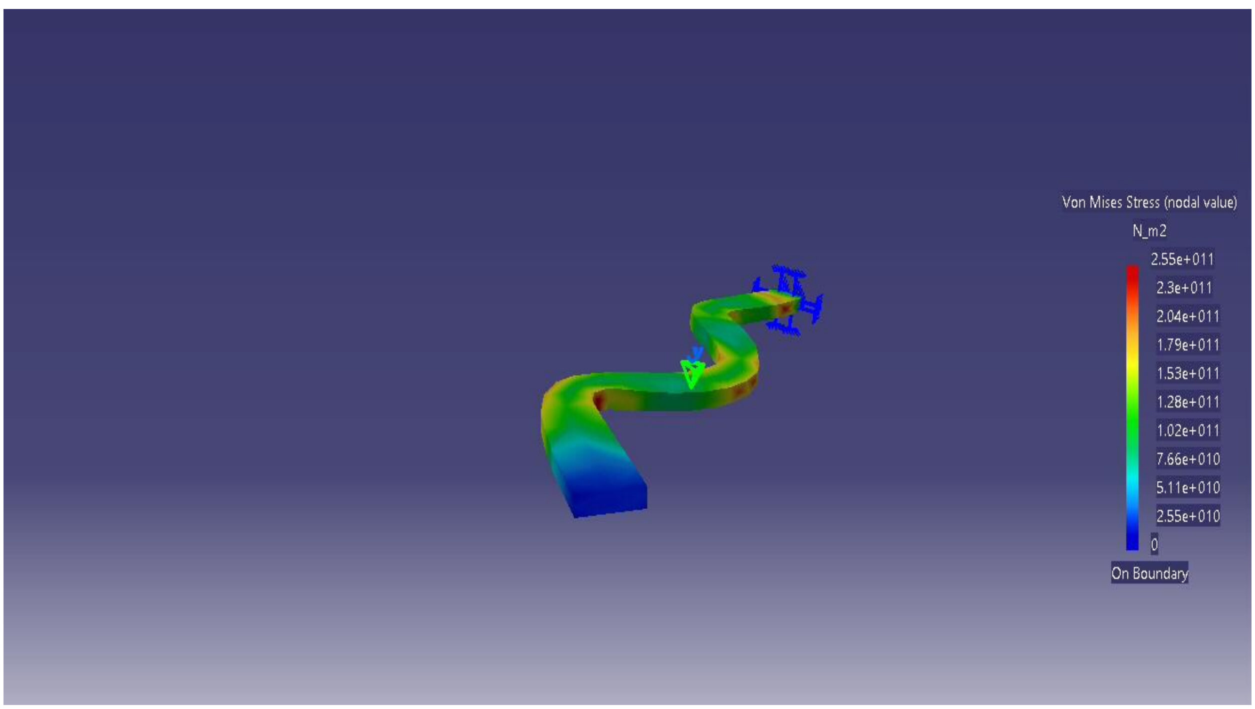

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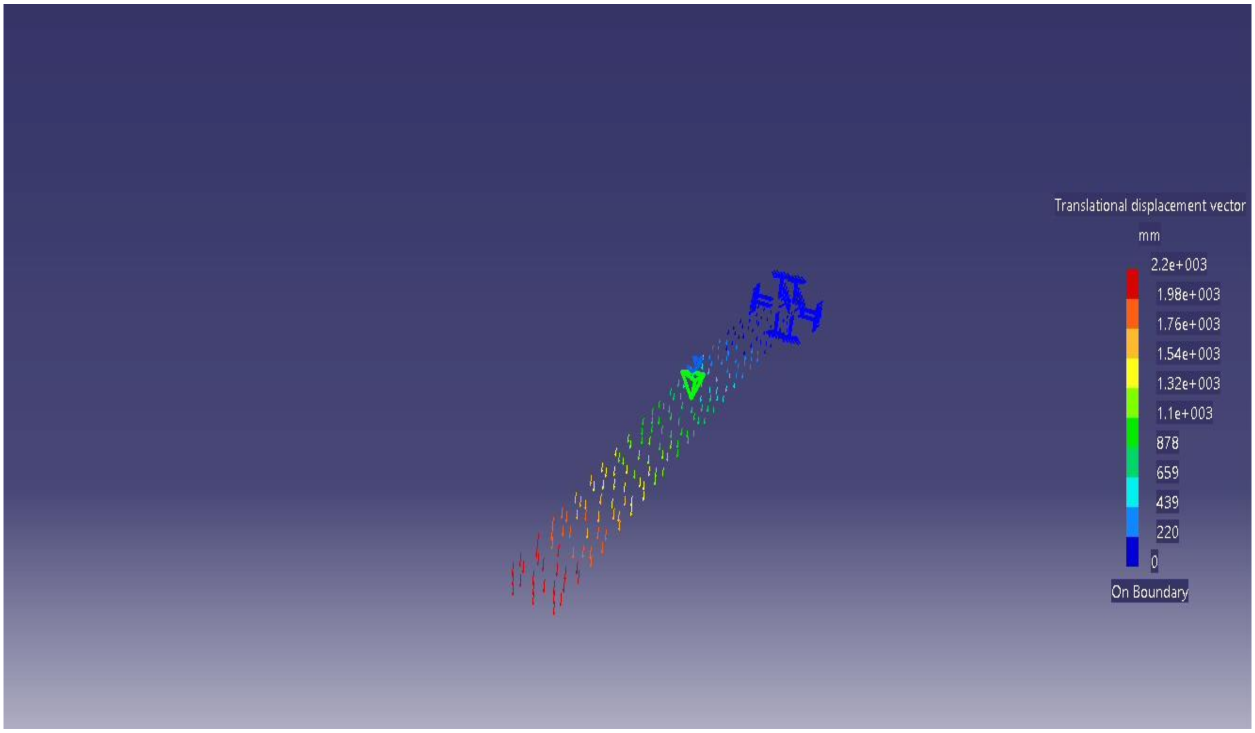
Figure 5.3: Von misses stresses at crack location 400 mm from the fixed end



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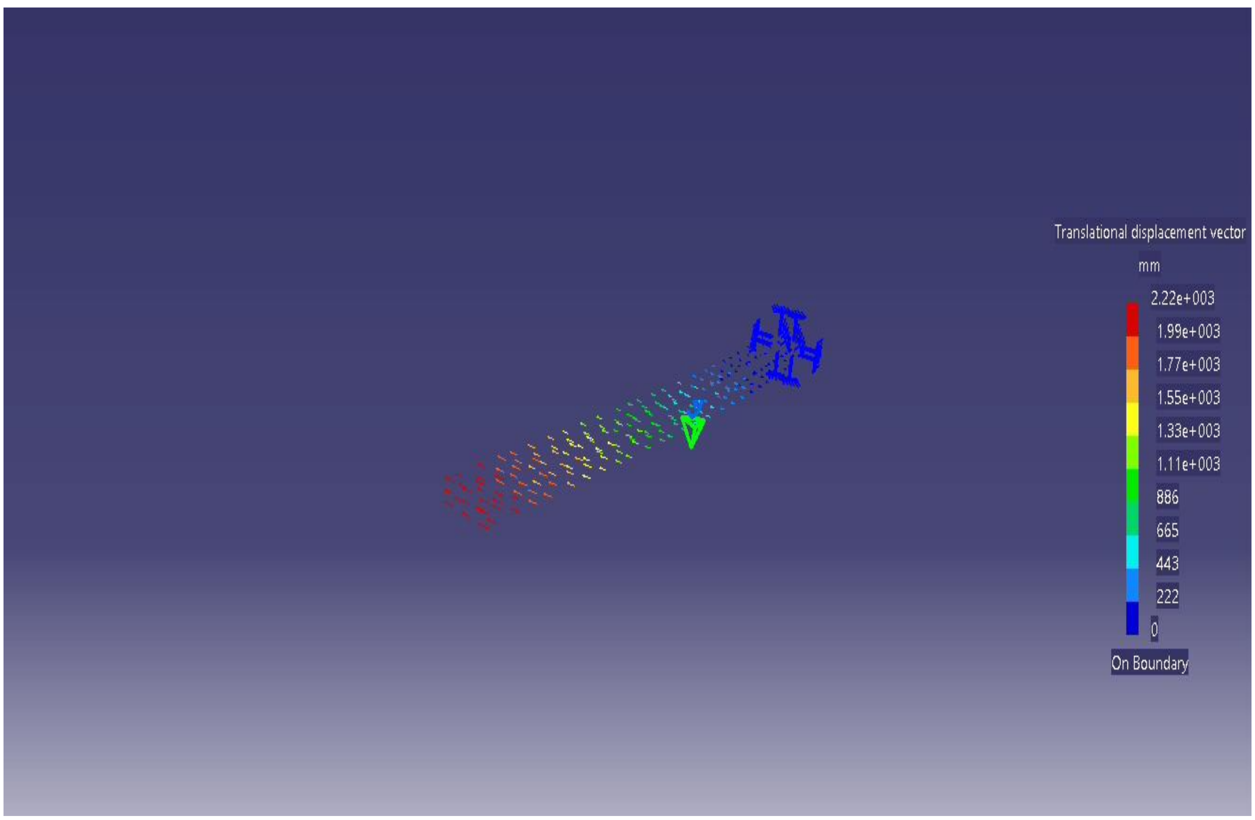

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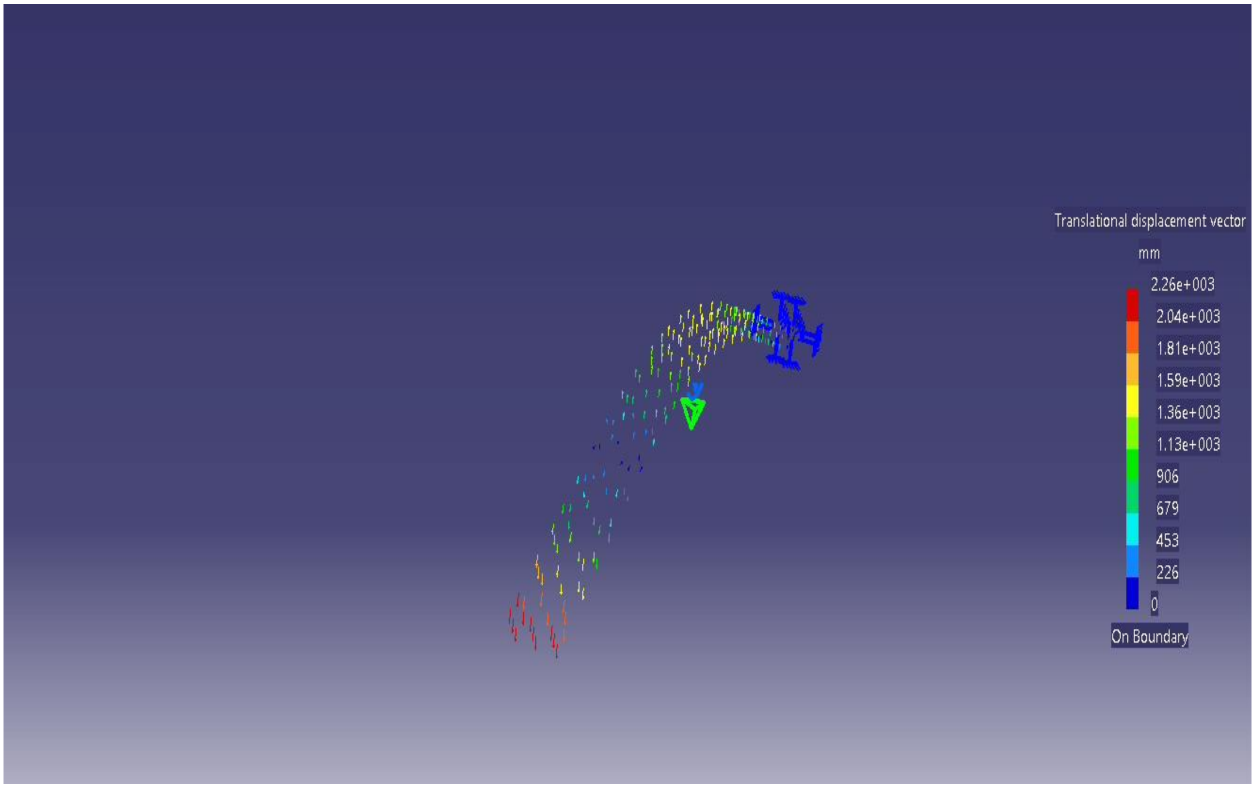

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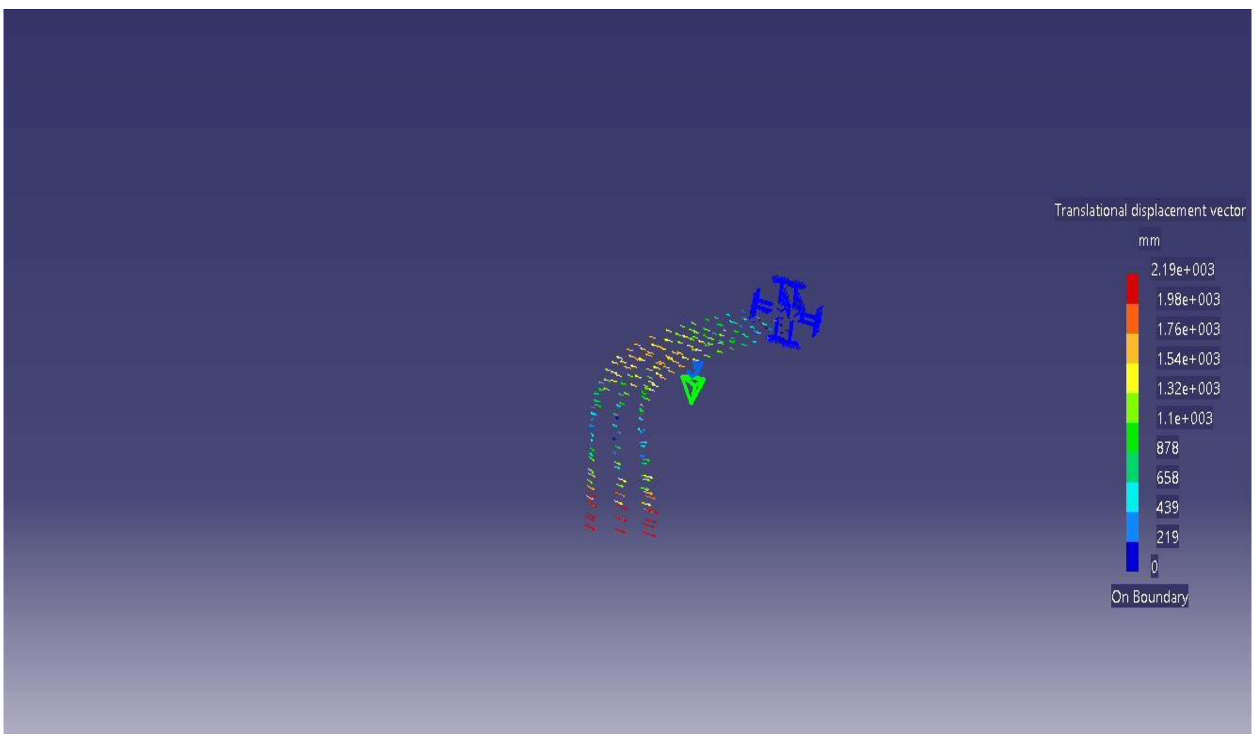

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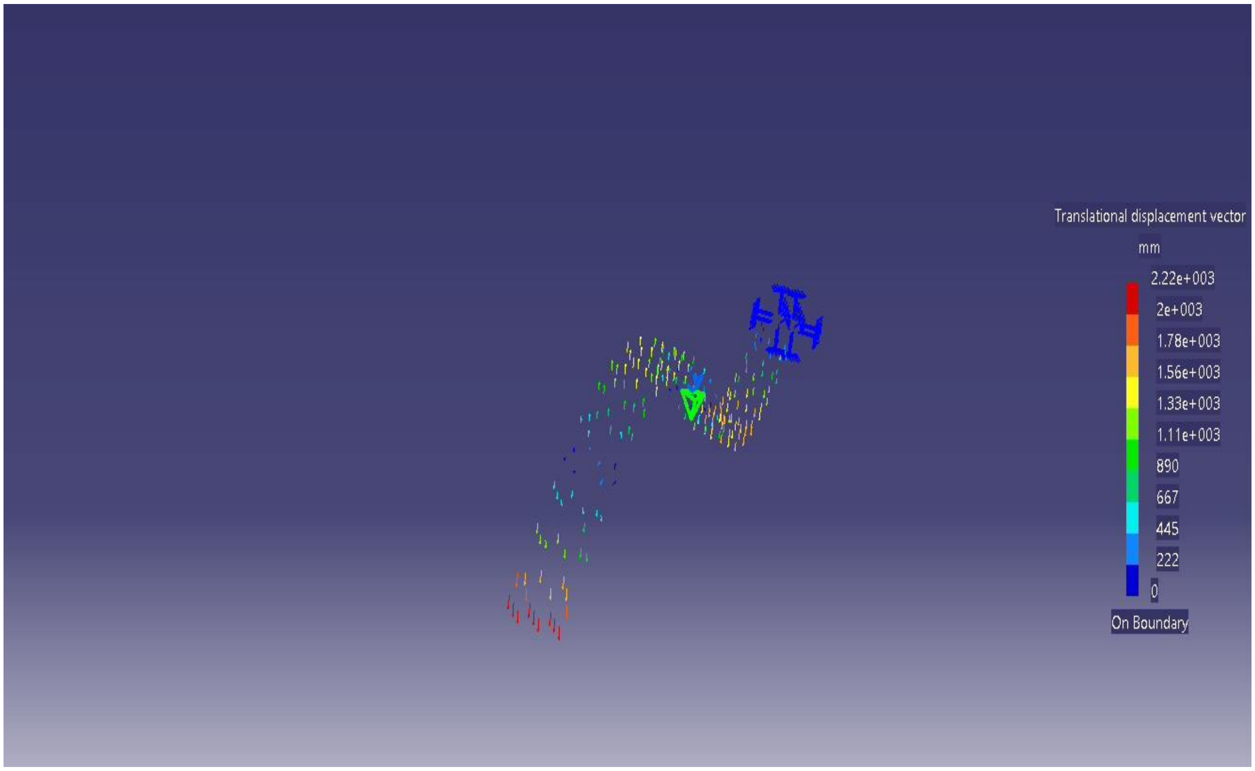

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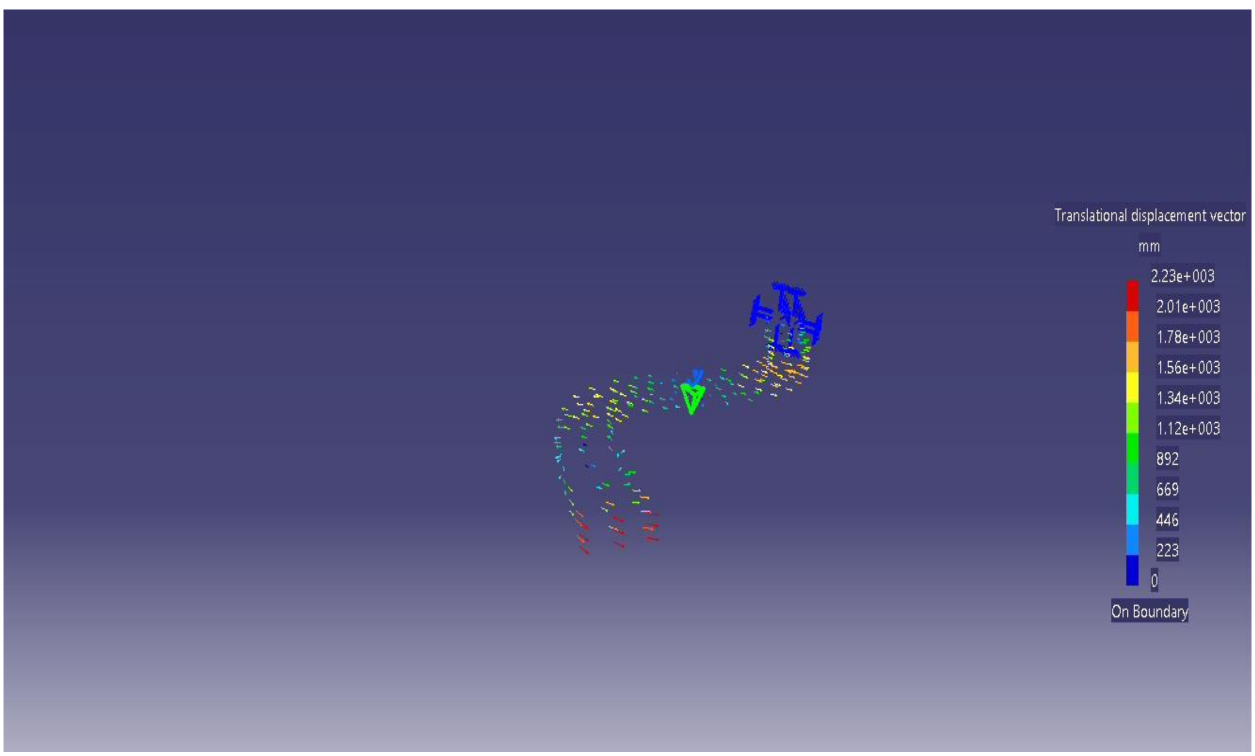

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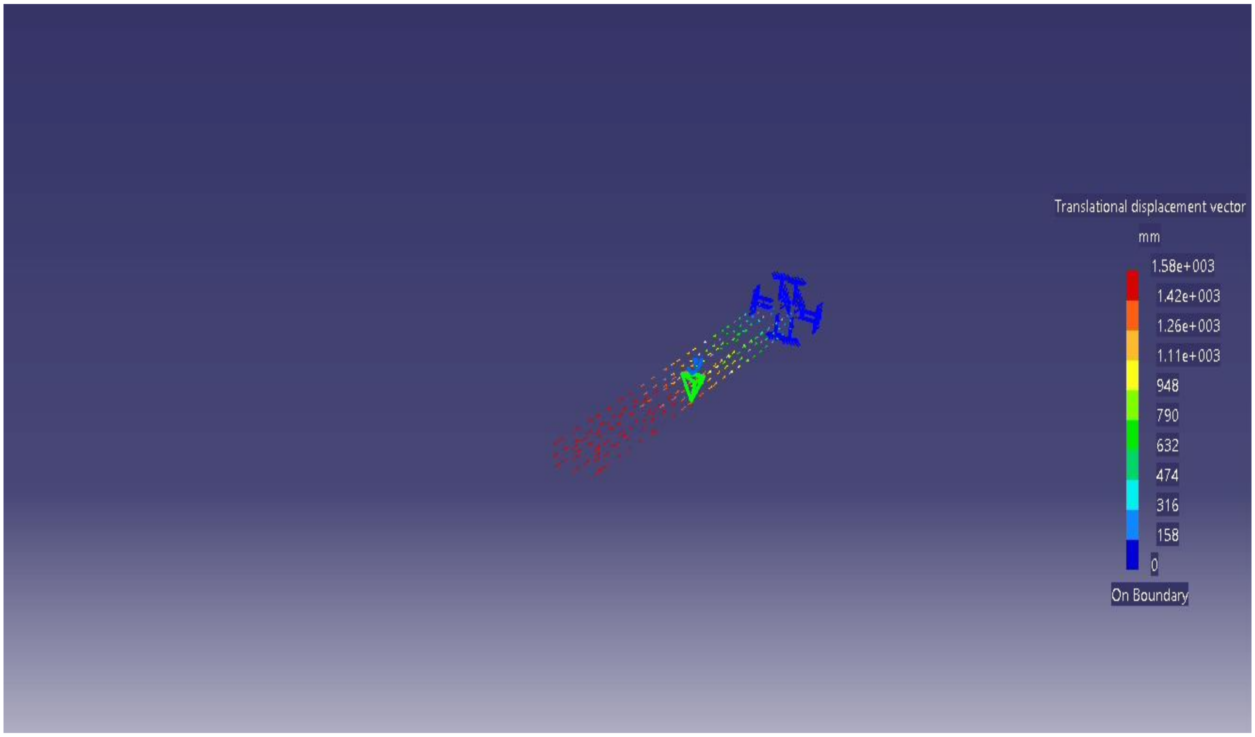

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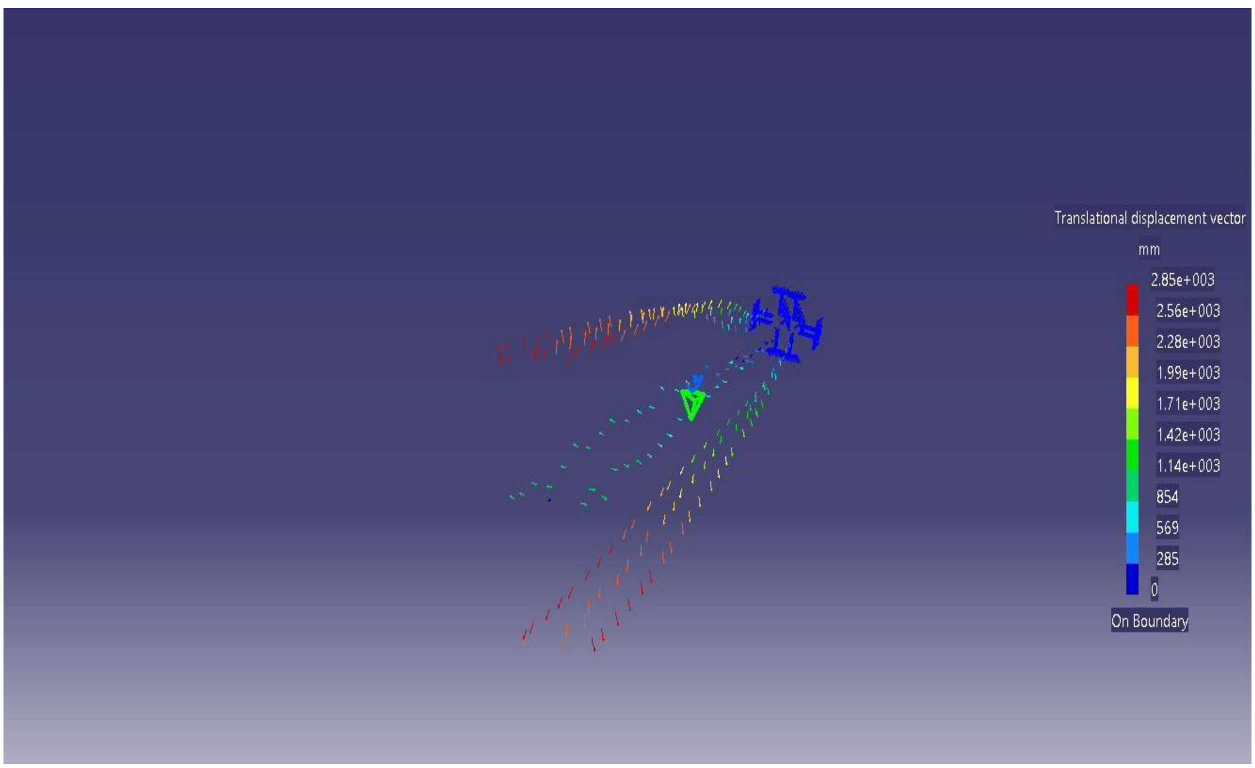

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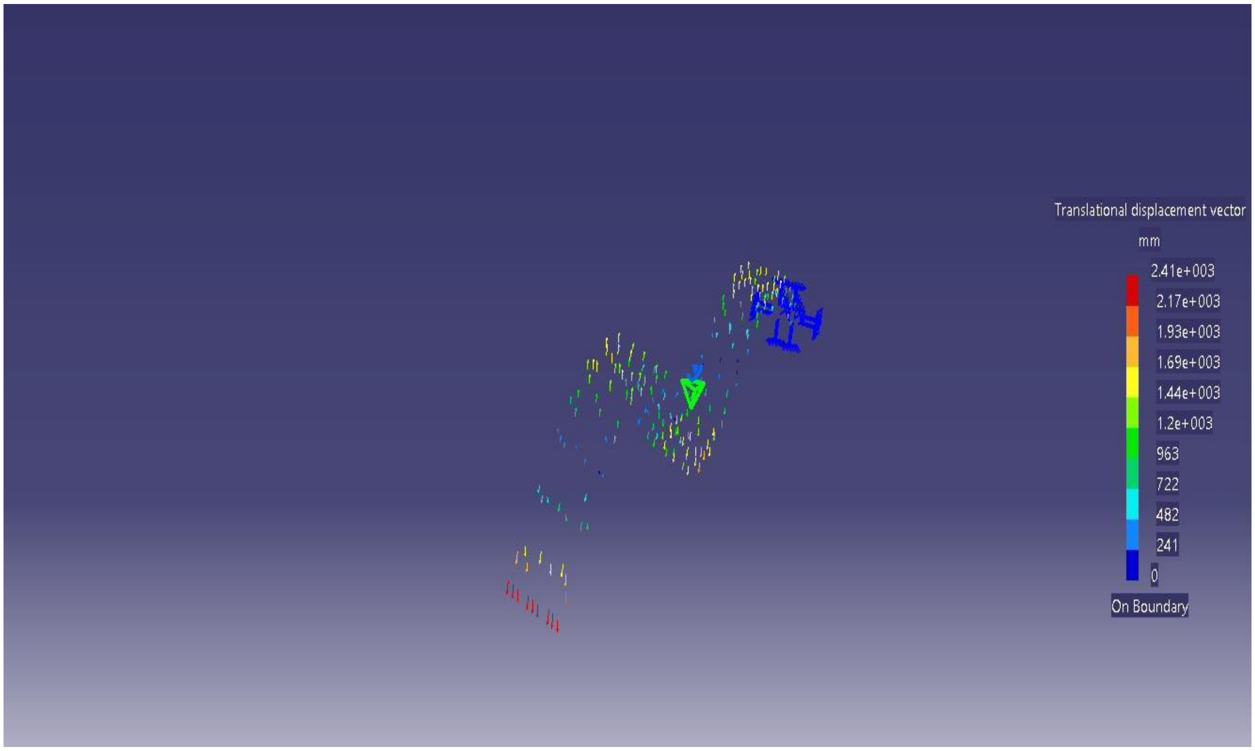

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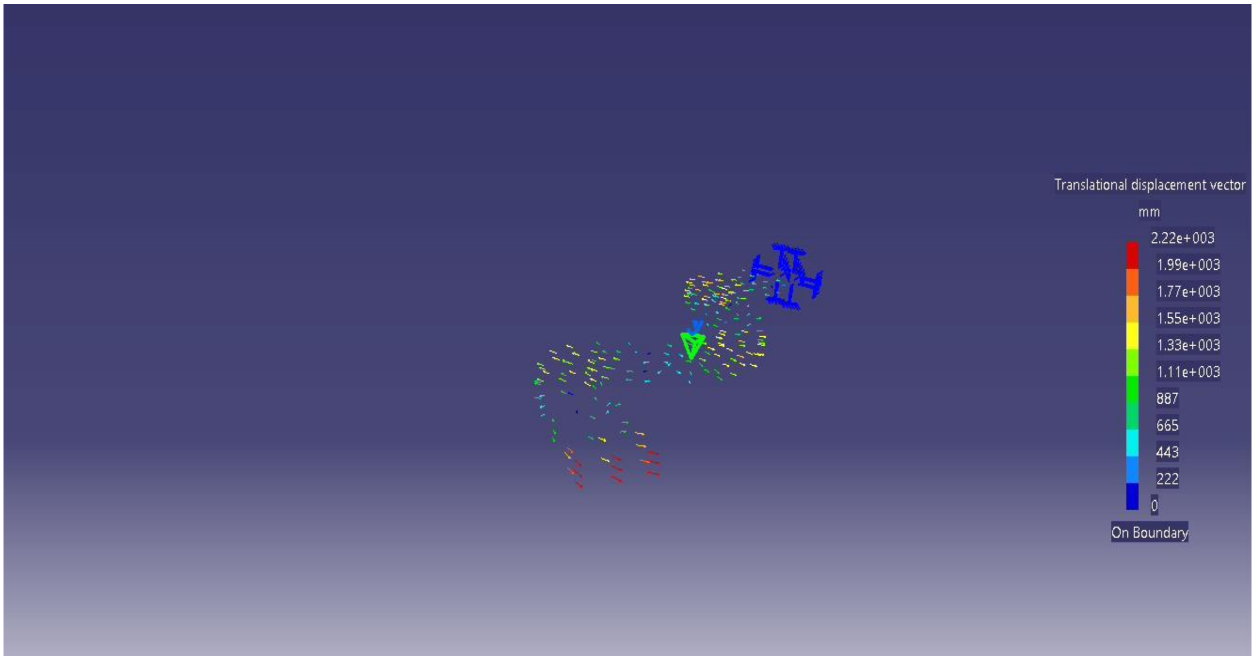

j) Mode 10

Figure 5.3: Displacements at crack location 400 mm from the fixed end

Table 5.1 presents the summary of results obtained from the modal analysis.

Table 5.1: Summary of results of Modal Analysis

S.No.	Mode Number	Frequency (Hz)		Von misses stresses (N/m^2)		Displacement (mm)	
		200 mm from free end	400 mm from free end	200 mm from free end	400 mm from free end	200 mm from free end	400 mm from free end
1.	1	45.7338	45.5639	6.46E+09	6.36E+09	2200	2200
2.	2	86.2116	86.0756	1.31E+10	1.32E+10	2220	2220
3.	3	291.865	297.24	3.38E+10	3.49E+10	2270	2260
4.	4	524.796	522.842	7.00E+10	7.01E+10	2200	2190
5.	5	810.678	811.465	7.00E+10	8.20E+10	2290	2220
6.	6	1393.58	1398.69	1.58E+11	1.59E+11	2280	2230
7.	7	1405.7	1404.35	1.07E+11	1.06E+11	1590	1580
8.	8	1445.87	1418.17	1.52E+11	1.48E+11	2930	2850
9.	9	1571.39	1615.91	1.39E+11	1.49E+11	2290	2410
10.	10	2574.44	2604.59	2.41E+11	2.55E+11	2310	2220

5.2 Discussion

The modal analysis of the cracked cantilever beam was performed for two crack locations—at 200 mm and 400 mm from the fixed end—to study variations in natural frequencies, von Mises stresses, and displacements for ten mode shapes.

The results show that the crack position significantly affects the beam's dynamic behavior. When the crack is closer to the fixed end (200 mm), slight reductions are observed in the first two natural frequencies, indicating reduced stiffness in the high bending moment region. However, the changes are small, suggesting that lower modes are less sensitive to local stiffness variations. As the mode number increases, the frequency difference between the two crack locations becomes more pronounced. For mid-range modes (3–6), variations up to 5 Hz were observed, while higher modes (7–10) exhibited larger differences, reaching up to 40–45 Hz. These findings confirm that higher-order modes are more sensitive indicators of crack location and severity, as they are associated with greater curvature and local deformation.

The von Mises stress distribution also varied with crack position and mode number. When the crack was near the fixed support, stresses were slightly higher in the lower modes due to the concentration of bending moments. As mode numbers increased, the stress differences became more evident, particularly in Modes 5 and 10, where stress values rose sharply for the crack at 400 mm. This suggests that cracks located away from the fixed end amplify local stresses at specific mode shapes where deflection peaks coincide with the crack region. Hence, higher modes provide more reliable insight into local stress concentrations, making them useful for identifying potential damage zones.

Displacement results followed a similar trend. For the first three modes, displacements were almost identical for both crack locations, confirming that global vibration patterns are not highly affected by moderate shifts in crack position. From Modes 4 to 7, minor reductions in displacement were observed at the 400 mm crack location, while higher modes (8–10) showed greater sensitivity with differences up to 120 mm. These variations reflect the localized flexibility and stiffness changes caused by the crack position.

Overall, the analysis demonstrates that crack location influences all three modal parameters—frequency, stress, and displacement—most notably in higher modes. Lower-order modes represent global beam behavior, while higher-order modes capture localized effects, making them valuable for crack detection and structural health monitoring.

6. Conclusion, Limitations and Future Scope of the Research

The present chapter focuses on conclusions, and limitations and future scope of the research work, the details of which are presented in upcoming sub-sections.

6.1 Conclusions

The following points represent the conclusions of the research work:

a) Crack location significantly influences the modal parameters (natural frequency, Von Mises stress, and displacement) of the cantilever concrete beam.

b) Lower-order modes (1–3) are relatively unaffected by crack location, showing only small variations in frequency, stress, and displacement, indicating that global behavior is dominated by overall stiffness rather than local damage.

c) Intermediate modes (4–6) display moderate variations, highlighting the interaction between crack position and vibration shape, especially in regions of higher curvature.

d) Higher-order modes (7–10) are the most sensitive to crack location, with large differences observed in frequency shifts, stress concentration, and displacement patterns.

e) A crack near the fixed end mainly influences global stiffness and fundamental behavior, while a crack further away (400 mm) affects higher modes, making them useful indicators for crack detection and localization.

f) The combined analysis confirms that modal parameters can be effectively used for structural health monitoring of cracked beams, with higher modes being better suited for damage localization.

6.2 Limitations and Future Scope of the Research

The following points represent the limitations of the research work:

a) The study considers only two crack locations (200 mm and 400 mm); a wider range of positions may provide deeper insights.

b) The crack is assumed to be of a specific size and depth; variation in crack severity is not covered in this analysis.

c) The analysis is performed on an idealized concrete beam model, neglecting real-world factors such as material heterogeneity, environmental effects, and damping.

d) The study is limited to free vibration (modal analysis) and does not include forced vibration or dynamic loading conditions, which may further affect beam behavior.

e) Experimental validation of the numerical results is not included, so the practical accuracy of the findings remains unverified.

The following points represent the future scope of the research work:

a) The study may be extended to include multiple crack locations and varying crack depths to simulate real-life conditions more accurately.

b) Experimental modal analysis may be performed on laboratory-scale cracked beams to validate the numerical results.

c) Damping and dynamic loading conditions may be incorporated to study real operational scenarios of cracked concrete beams.

d) Advanced numerical methods such as fracture mechanics models or nonlinear analysis may be applied to capture progressive crack propagation effects.

e) A predictive model or damage detection algorithm that uses frequency, stress, and displacement variations for automated crack detection in civil structures may be developed.

f) Applications of non-destructive testing (NDT) techniques in combination with modal analysis for more robust structural health monitoring may be explored.

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